Bonds vs. Equities: Information for Investment*

Huifeng Chang†, Adrien d'Avernas‡, Andrea L. Eisfeldt§

November 17, 2021

Abstract

Why do bond market measures of risk, such as credit spreads, perform better at predicting real economic outcomes and recessions than equity market measures of risk, such as equity volatility? To answer that question, we provide robust empirical evidence that equity volatility is an ambiguous signal for real economic activity. Using firm-level data, we find that the sensitivity of investment to equity volatility is highly significant, but changes sign in the cross section of firms depending on their distance to default. This sign change confounds aggregate inference. We rationalize these findings using a simple structural model of credit risk and investment with debt overhang.

*We would like to thank participants in the Macro Finance Lunch at UCLA, the Stockholm School of Economics Finance Brown Bag, the NBER Summer Institute, the Macro Finance Society Workshop and Vincent Maurin for helpful comments.

†UCLA Economics, email: huifengchangpku@gmail.com

‡Stockholm School of Economics, email: adrien.davernas@gmail.com

§UCLA Anderson School of Management and NBER, email: andrea.eisfeldt@anderson.ucla.edu

1 Introduction

Economists and practicioners alike have long argued that there is a tight connection between bond markets and the macroeconomy. Friedman and Kuttner (1992) show that the spread between commercial paper and Treasury bills forecasts recessions. Gilchrist and Zakrajšek (2012) use firm-level data to construct a credit spread measure with substantial predictive power for consumption, inventories, and output. Philippon (2009) constructs Tobin's q from bond market data and shows that it outperforms equity-market q in predicting firm-level investment.[1]

An open question, however, is why bond market data appears to have better forecasting power for real outcomes (in particular during recessions) than equity market data. Is it because bond markets have more "smart money" or better reflect changes in financial market distortions? Or because bonds capture downside risk better, while equity prices are more affected by growth options? In this paper, we study the relationship between equity volatility and credit spreads—the two most widely-used measures of risk based on equity and bond markets data—and provide robust empirical evidence and a model of investment with debt overhang that support the latter explanation.

In particular, we focus on the investment channel. Bloom (2009) shows that shocks to uncertainty measured using implied equity volatility forecast lower investment.[2] However, recent work by Gilchrist, Sim, and Zakrajšek (2014) indicates that controlling for credit spreads substantially reduces the predictive power of equity volatility for investment.[3] This is surprising since bond spreads and equity volatility are tightly related. They both reflect a combination of firms' asset volatilities and leverage and the (the inverse of) equity volatility and credit spreads contain very

[1] See also the important contributions by Friedman and Kuttner (1998), Bernanke (1990), Gertler and Lown (1999), and Gilchrist, Yankov, and Zakrajšek (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2014), Krishnamurthy and Muir (2017).

[2] See Panousi and Papanikolaou (2012) for related evidence showing that the negative relation between idiosyncratic equity volatility and investment is stronger when managerial ownership is higher.

[3] Gilchrist, Sim, and Zakrajšek (2014) suggest that uncertainty makes investment more costly by increasing firms' credit spread. See also Christiano, Motto, and Rostagno (2014) and Arellano, Bai, and Kehoe (2019).

similar information about firms' financial soundness.[4]

To shed light on that puzzle, we establish four main empirical facts. First, as documented by Gilchrist, Sim, and Zakrajšek (2014), credit spreads drive out equity volatility in an empirical model of the sensitivity of firm-level investment to equity volatility and credit spreads. But, this result is due to systematic heterogeneity in the sensitivity of investment to equity volatility in the cross section of firms. The sensitivity of investment to equity volatility is positive for firms far enough away from default, and negative otherwise. These different signs in the cross section drive the pooled effect of equity volatility to be less significant than credit spreads. By contrast, the elasticity of investment to credit spreads is always negative.

Second, using fair value spreads, we repeat the above analysis and show that the results are virtually identical. These fair value spreads are constructed using structural models of credit risk which derive credit spreads from asset volatility and leverage. They are spreads measured without any bond market data which capture only the credit risk portion of bond spreads.[5] Thus, the different information in equity volatility and credit spreads for investment we document cannot be due to bond markets having more "smart money". It also cannot be due to bond spreads better reflecting changes in financial market distortions.

Third, the levels of both equity volatility and credit spreads are in large part driven by asset volatility and leverage, as predicted by structural models of credit risk.[6] However, credit spreads have higher loadings on leverage, while equity volatility loads more on asset volatility. This is intuitive given the priority of debt versus equity in firms' capital structures and, together with our fourth fact, is helpful for understanding why equity volatility might positively impact investment decisions.

Fourth, the sensitivity of investment to *asset* volatility is positive for all firms.

[4]Based on the the seminal work of Merton (1974) and Leland (1994), Atkeson, Eisfeldt, and Weill (2017) show that one can approximate a firm's distance to insolvency using the inverse of equity volatility.

[5]See Arora, Bohn, and Zhu (2005) and Nazeran and Dwyer (2015).

[6]Collin-Dufresn, Goldstein, and Martin (2001) show that *changes* in credit spreads also have a common component that appears unrelated to structural determinants, however we show the majority of variation in credit spread levels, and about one third of credit spread changes, can be explained by asset volatility and financial leverage.

Two interpretations of that novel result are possible. First, the uncertainty of future investments feeds back into the volatility of current asset values (what we call the endogeneity channel). Second, asset volatility boosts the option value of equity, dilutes the debt overhang effect, and incentivizes equity holders to invest more (what we call the causal channel). We chose these explanations because they provide useful benchmarks. They are not mutually exclusive, and they are not the only possible explanations.

We provide four empirical exercises to assess which channel is driving our results: (i) a Granger causality test, (ii) whether our results are stronger for firms with more intensive research and development or that invest more, (iii) whether our results are stronger for firms that have tighter debt covenants, and (iv) the instrumental variables strategy of Alfaro, Bloom, and Lin (2018) to address endogeneity in estimating the impact of equity and asset volatility on investment. The results of the four tests are consistent with the causal channel of option value driving our results.

We build a simple model of investment to study the option value of asset volatility and debt overhang channel of credit spreads. Because of the debt overhang effect, equity holders choose a suboptimal level of investment. An increase in asset volatility has the potential to boost the option value for equity holders as, in the presence of debt with limited liability, they face limited downside but unlimited upside. We show that, controlling for credit spreads, an increase in asset volatility always has a positive effect on investment. In contrast, equity volatility is an ambiguous signal for investment, as an increase in equity volatility can reflect an increase in leverage or an increase in asset volatility, which have opposite impacts on equity holders' incentives to invest. Interestingly, we show that controlling for asset volatility and leverage instead of asset volatility and credit spreads also leads to asset volatility being an ambiguous signal for investment, a prediction of the model that we confirm in the data.

[Figures 4 and 5 about here.]

To document the importance of our findings for understanding the role of uncertainty and credit spreads on aggregate activity, we plot the time series and cross section of the estimated firms' elasticity of investment with respect to equity volatility in Figure 4. Firms with lower credit spreads which are further away from default display a positive elasticity of investment, while firms with higher credit spreads display a negative elasticity. Aggregate effects are driven by the movement of the entire cross section of firms away from and closer to their respective default boundaries. Thus, a positive shock to equity volatility has a particularly dire impact on investment when the entire cross section of firms is closer to default. In contrast, Figure 5 shows that the elasticity of investment to credit spreads is negative for all firm-quarters. We also confirm that our micro-results aggregate with a recursive vector autoregression model of the aggregate time series of investment, asset volatility, and credit spreads. As expected, the aggregate investment response to a positive shock to asset volatility is positive while the response to a positive shock to credit spreads is negative.

The remainder of the paper is organized as follows. Section 2 discusses data sources and the construction of variables. Section 3 presents our firm-level empirical results. In Section 4, we show that our results hold at the aggregate level. Section 5 presents our model to build economic intuition. Section 6 concludes.

2 Data and Definitions

This section discusses the data sources used for the empirical analysis and the construction of variables.

Data Collection We use S&P's Compustat quarterly database from 1984:Q1 to 2018:Q4. We exclude firms in the financial sector (SIC code 6000 to 6999) and utility sector (SIC code 4900 to 4949), firms not in the panel for at least 3 years, and observations with missing investment rate, equity volatility and with negative sales. We use daily returns from the Center for Research in Security Prices (CRSP) database. Bond prices come from the Lehman/Warga (1984-2005) and ICE databases (1997-2018). This selection criterion yields 1,273 unique firms with 42,580 firm-quarter observations. To ensure that our results are not driven by extreme values, we trim every regression variables at the 1 and 99 percentiles. We provide summary statistics in Table 1 and describe how we construct our key variables below.

[Table 1 here.]

Investment and Equity Volatility We define investment rate as capital expenditures in quarter t scaled by net property, plant, and equipment in quarter $t-1$. Idiosyncratic equity volatility is constructed in two steps. For each firm-fiscal quarter, we extract daily excess returns using the Carhart (1997) four-factor model. Then for each regression we calculate the standard deviation of residuals over one quarter, and obtain quarterly firm-specific idiosyncratic equity volatility. We only keep observations for quarters with more than 30 trading days. As an alternative measure of equity volatility, we also use at-the-money 30-day forward put options implied equity volatility from OptionMetrics.

Credit Spreads We follow Gilchrist and Zakrajšek (2012) to compute bond-level credit spreads. First, we construct a theoretical risk-free bond that replicates exactly the promised cash flows. The price of this risk-free bond is calculated by discounting the promised cash flows using continuously-compounded zero-coupon Treasury yields from Gürkaynak, Sack, and Wright (2007). The credit spread of an individual bond is the difference between the yield of the actual bond and the yield of the corresponding risk-free bond. We then define the credit spread of a firm as the average of the quarter-end credit spreads of all bonds issued by that firm.

Market Leverage Market leverage is defined as the ratio of market value of assets to market value of equity. The market value of assets is built as the book value of assets plus the market value of equity minus the book value of equity. Following Davies, Fama, and French (2000), the book value of equity is defined as the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit, minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) for the book value of preferred stock. If this procedure generates missing values, we measure stockholders' equity as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities.

Return on assets, Tangibility, Sales, Income, and Tobin's q Return on assets is operating income before depreciation divided by total assets. Tangibility is property, plant and equipment divided by total asset. The sales and income ratios are given by sales and operating income before depreciation divided by lagged property, plant and equipment. Following Erickson and Whited (2012), we construct the numerator of Tobin's q as book debt plus market value of equity minus book assets while the denominator is capital stock.

Asset Volatility We construct a measure of firm-level idiosyncratic asset volatility based on Merton's (1974) model. We implement the iterative procedure proposed by Bharath and Shumway (2008), and then use the resulting asset values to generate times series of daily asset returns. With time series of daily asset returns, we calculate the idiosyncratic asset volatility using the same methodology used for idiosyncratic equity volatility. In addition to this *realized* asset volatility measure, we also use an *implied* asset volatility measure. Implied asset volatility is constructed as delevered implied equity volatility, that is, implied equity volatility times market value of equity divided by market value of assets.

Fair Value Spreads We use proprietary data set from Moody's on its Public Firm Expected Default Frequency (EDF) Metric, which is an equity-based measure of firm's probability of default. The core model used to generate the EDF metric belongs to the class of option-pricing based, structural credit risk models pioneered by Black and Scholes (1973) and Merton (1974). The Vasicek-Kealhofer (VK) model summarizes information on asset volatility, market value of assets, and the default point into one metric, distance-to-default (DD), and then maps the DD to obtain the EDF metric. The DD-to-EDF mapping step utilizes the empirical distribution of DD and frequency of realized defaults. Nazeran and Dwyer (2015) provide a detailed description of their methodology. Most importantly for our purpose, the EDF credit risk measure relies only on equity market inputs and does not contain bond market information.

Using the EDF credit risk measure, we construct a cumulative EDF (CEDF) over T years by assuming a flat term structure, that is, $CEDF_T = 1-(1-EDF)^T$. Then, we convert our physical measure of default probabilities (CEDF) to risk-neutral default probabilities (CQDF) using the following equation:

$$CQDF_T = N\left[N^{-1}\left(CEDF_T\right) + \lambda\rho\sqrt{T}\right],$$

where N is the cumulative distribution function for the standard normal distribution, λ is the market Sharpe ratio and ρ is the correlation between the underlying asset returns and market returns. Given this risk-neutral default probability measure, the spread of a zero-coupon bond with duration T can be computed as:

$$\hat{s} = -\frac{1}{T}\log(1 - CQDF_T \cdot LGD),$$

where LGD stands for the risk-neutral expected loss given default. We follow Moody's convention and set $T = 5$, $LGD = 60\%$, $\lambda = 0.546$, and $\rho = \sqrt{0.3}$ to build our "fair value spread" measure $\hat{s}$. We successfully match $39,925$ fair value spreads with our firm-quarter observations.

Covenant Tightness To measure the strength of creditor control rights, which is useful for providing empirical support for the debt overhang channel in our model, we use a covenant tightness measure based on a firm's outstanding loans. Data on covenant specifications and thresholds for loans is from DealScan. There are 18 types of covenants in the data. We first compute the distance between the actual financial ratio and the covenant threshold for each type of covenant, normalized by the firm-specific standard deviation of the actual financial ratios. We then use the minimum of the normalized distances to measure the overall covenant tightness for the firm in each quarter. See Kermani and Ma (2020) for more details on the covenant tightness measure.[7]

3 Firm-level Panel Regressions

In this section, we present a set of firm-level panel regressions of investment rate on volatility and spreads:

$$\log[I/K]_{i,t} = \beta_1 \log X^{\sigma}_{i,t-1} + \beta_2 \log X^{dd}_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}, \tag{1}$$

where $\log[I/K]_{i,t}$ is the log of investment rate of firm i in period t, $X^{\sigma}_{i,t-1}$ denotes measures of volatility (idiosyncratic equity volatility $\sigma^{e}_{i,t-1}$ or idiosyncratic asset volatility $\sigma_{i,t-1}$), and $X^{dd}_{i,t-1}$ denotes measures of distance-to-default (credit spread $cs_{i,t}$, fair value spreads $\hat{s}_{i,t-1}$, or market leverage $[MA/ME]_{i,t-1}$), all lagged by one quarter. We control for the firm fixed effects and time fixed effects by including η_i and λ_t. Following Alfaro, Bloom, and Lin (2018), our control variables $\mathbf{X}_{i,t-1}$ include the lag of firm i's return on equity, log tangibility, log sale ratio, log income ratio, and log Tobin's q.

Equity Volatility and Credit Spread We first replicate the results in Gilchrist, Sim, and Zakrajšek (2014) that the adverse effect of idiosyncratic equity volatility on investment is dampened when controlling for credit spreads. Table 2 presents the estimation results of equation (1) using equity volatility and credit spread as control variables.

[Table 2 here.]

As shown in columns 1-3 of Table 2, the coefficient on idiosyncratic equity volatility and credit spread are statistically significant and economically important on their own (columns 1-2). However, when both measures are included in the regression,

[7]We thank Yueran Ma and Amir Kermani for sharing their data with us.

the coefficient on equity volatility is substantially reduced both in terms of magnitude and statistical significance while the coefficient on credit spread is unaffected (column 3).

To see why bond spreads can drive out equity volatility, we sort firms into tercile groups based on credit spread each quarter and run the same regression for these subsamples (columns 4-6).[8] We find that the coefficient on equity volatility changes sign in the cross section: it is significantly positive among firms with low credit spread and significantly negative among firms with high credit spread. The last column shows results from the regression with an interaction term and confirms our findings from columns 4-6. A simple back-of-the-envelope calculation suggests that the sign flip happens at a credit spread level of 194 basis points.

[Table 3 here.]

In Table 3, we replace credit spreads with fair value spreads. The results are qualitatively identical to Table 2. The coefficient on equity volatility goes from significantly positive to significantly negative as firm's credit spread goes up, while the coefficient on the fair value spread remains significantly negative across the subgroups. As the fair value spreads are constructed with only equity market information and does not contain bond market information, the results from Table 3 cannot be driven by differences in the investor base or information about financial frictions only reflected in credit spreads.

Asset Volatility and Credit Spread Equity volatility can be decomposed into asset volatility (derived from Merton's model) and market leverage. In Table 4, we run the same regression but we replace idiosyncratic equity volatility $\sigma^e_{i,t}$ with idiosyncratic asset volatility $\sigma_{i,t}$. The coefficient on asset volatility is always positive and statistically significant in the full sample and in all subgroups. Interestingly, the positive impact of asset volatility on investment is statistically stronger for firms with lower credit spreads, while the reverse is true for credit spreads.

[8]This method of splitting uses quarter-specific cutoffs. Using fixed cutoffs to sort all firm-quarter observations leads to similar results.

[Table 4 and Table 5 here.]

In our model, equity holders make investment decisions given uncertainty about future returns. In Table 5, we replicate the same exercise but with implied asset volatility from equity options. The results are stronger, both economically and statistically, than the results with volatility derived from past equity return observations, lending support to the idea that it is the expectation of future asset volatility that drives changes in investment, not past uncertainty.

[Table 6 here.]

Given the decomposition of equity volatility, a natural question is whether the coefficient on implied asset volatility is also positive when market leverage is used as an additional explanatory variables for investment. As shown in table 6, the coefficient on implied asset volatility changes sign in the cross-section when the level of credit spread is not controlled for. In the model section, we rationalize this finding by showing that, together, leverage and asset volatility are not unambiguous signals of debt overhang and option value.

Loadings Asset volatility and leverage are also important drivers for bond spreads. To understand why there is no such sign flip for credit spreads, we consider the loadings of credit spreads and equity volatility on asset volatility and leverage and estimate the following equation:

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

where $y_{i,t}$ is either the equity volatility ($\sigma^e_{i,t}$) or the credit spread ($s_{i,t}$), and $MA/ME_{i,t}$ is market leverage. We estimate the equation both in levels and in first differences.

[Table 7 here.]

Table 7 summarizes the results. Columns 1-2 show how the levels of equity volatility and credit spread load on levels of asset volatility and leverage, and columns 3-4 show how the changes load on corresponding changes. Both specifications imply that changes in bond spreads are mainly driven by leverage, while changes in equity volatility are driven by both asset volatility and leverage. Since shocks to asset volatility and leverage impact investment differently, bond spreads and equity volatility contain different information for investment. In Appendix C, instead of using the firm's asset volatility and market leverage directly, we use the industry-level regressors, constructed as a simple average of all firms in the same industry excluding the firm itself. This exercise shows similar patterns that equity volatility loads more on asset volatility while credit spread loads more on leverage.

Thus, an increase in equity volatility could either signal an increase in asset volatility (positive for investment) or in leverage (negative for investment). Whether one force dominates the other changes in the cross-section. As seen in Table 4, the asset volatility effect weakens as firms' credit spreads increase, while the leverage effect strengthens.[9] Thus, the sensitivity of investment to equity volatility becomes negative when the leverage effect dominates for firms with higher credit spreads. Although credit spreads are also a combination of asset volatility and leverage, the loading of credit spreads on asset volatility is not large enough to ever drive a positive relation between credit spreads and investment.

3.1 Endogeneity

We hypothesize that the positive correlation between investment and asset volatility is most likely driven by two mechanisms: (i) due to higher investments, the value of the assets of the firm become more uncertain and (ii) an increase in business risk makes the value of assets in place more volatile and incentivizes firms to invest more. In this section, we document four tests that lend support for the later mechanism dominating the former.

Lags and Investment Intensity If asset volatility increases because of uncertainty driven by more investment in place, we would expect that (i) an increase in

[9]The statistical significance of the credit spreads coefficient in Table 4 but also the leverage coefficient in Table 6 strengthens when credit spreads are higher.

investment to Granger-cause an increase in asset volatility and (ii) that the correlation is more pronounced for firms with higher levels of research and development or investment. Table 8 shows that the coefficients on asset volatility are statistically and economically larger to explain the variation in investment with lags rather than leads. Table 9 shows that our results are weaker for firms with higher levels of research and development or investment. Thus both exercises suggest that this potential mechanism is not driving our results.

[Tables 8 and 9 here.]

Instrumental Variables We use the instrumentation strategy of Alfaro, Bloom, and Lin (2018) to address endogeneity in estimating the impact of equity and asset volatility on investment. First, we estimate sensitivities to energy, currencies, treasuries, and policy at the industry level as the factor loadings of a regression of a firm's daily stock return on the price growth of energy, 7 currencies, return on treasury bonds, and changes in daily policy uncertainty. That is, for firm i in industry j, the sensitivity β_j^c is estimated as follows:

$$r_{i,t} = \alpha_j + \sum_c \beta_j^c \cdot r_t^c + \varepsilon_{i,t},$$

where $r_{i,t}$ is the daily risk-adjusted return on firm i, r_t^c is the change in the price of commodity c, and α_j is industry j's intercept.

The risk-adjusted returns $r_{i,t}$ are the residuals from running firm-level time-series regressions of daily CRSP stock returns on the classical Carhart (1997) four-factor asset pricing model. When constructing instruments for equity volatility, we use equity risk-adjusted returns, while we use delevered equity risk-adjusted returns when constructing instruments for asset volatility.

Finally, for these 10 aggregate market price shocks (oil, 7 currencies, treasuries, and policy) we multiply the absolute value of their time-varying sensitivities $|\beta_j^c|$ by shocks to their implied volatilities σ_t^c. This provides 10 instruments for lagged firm-level uncertainty, as follows:

$$z^c_{i,t-1} = |\beta^c_j| \cdot \sigma^c_{t-1}.$$

We refer the reader to Alfaro, Bloom, and Lin (2018) for further details on the construction of the instrumental variables.

Importantly, our results—a higher level of asset volatility is associated with a higher level of investment—are compatible with the results of Alfaro, Bloom, and Lin (2018)—a positive shock to uncertainty is associated with a lower level of investment. Indeed, while firms might postpone their investment following a large temporary uncertainty shocks, they can still invest more when the level of uncertainty is persistently higher. We illustrate that result in Table 10 by adding the shock to implied asset volatility $\Delta \log(\widehat{\sigma}_{i,t-1}) = \log(\widehat{\sigma}_{i,t-1}) - \log(\widehat{\sigma}_{i,t-2})$ as an additional control variable similar to the uncertainty shock of Alfaro, Bloom, and Lin (2018).

[Table 10 here.]

In Table 11, we show the results of our instrumental variable regression. While the estimated coefficient are statistically less significant, our main results hold: equity volatility has a positive impact on investment conditional on low credit spreads and asset volatility has an unconditional positive impact on investment.

[Table 11 here.]

Covenant Tightness As we are attributing the main force driving our results to debt overhang, we expect this coefficient to be stronger when debt holders have tighter control over cash flows resulting from investment. To test this hypothesis, we split the sample into two groups according to its covenant tightness following the measurements of Kermani and Ma (2020). The observations with the overall measure of distance between actual financial ratios and covenant thresholds below median are placed in the "Tight Covenant" group, and the remaining are assigned to the "Slack

Covenant" group. We estimate equation 1 with the interaction term using the two subsamples. The results are summarized in Table 12. For the subsample with tight covenant, all the coefficients are larger in absolute value, and are statistically more significant. In particular, the positive coefficient for equity volatility is twice as large for firms with tight covenant. This exercise provides empirical support for our model with debt overhang as a key distorting force.

[Table 12 here.]

4 Aggregates

Time Series To understand the implications of our findings for time series, we plot the elasticity of investment rate with respect to equity volatility, asset volatility, and credit spread across time and across firms using the estimates from the regressions with interaction terms. In Figure 4, we compute the overall coefficient on equity volatility at each credit spread level using estimates on equity volatility ($\log \sigma^e_{i,t}$) and the interaction term ($\log \sigma^e_{i,t} \times \log s_{i,t}$) reported in the last column of table 2. We repeat the procedure for asset volatility and credit spreads in **??** and Figure 5. Figure 4 shows that the cross section of elasticities of investment with respect to equity volatility varies a lot over time. In particular, this coefficient is negative for the whole cross-section of firms during the Great Recession, while it is mainly positive in the late 1980s. By contrast, in Figures **??** and 5, the elasticity of investment to asset volatility remains positive and the elasticity to credit spread remains negative, both in the cross-section and over time.

VAR Analysis Using an identified vector autoregression (VAR) framework, we confirm that our micro-level result—asset volatility has positive impact on investment—still holds at the macro-level. We aggregate the variables in our sample and estimate a simple VAR consisting of the three endogeneous variables: the log of idiosyncratic asset volatility ($\log \sigma_t$), the log of credit spread ($\log s_t$), and the log of investment rate ($\log[I/K]_t$).[10] We employ a standard recursive ordering technique and consider two identification schemes, one in which credit spread has an immediate impact on asset volatility and the other where asset volatility has immediate impact on credit spread.

[Figure 6 here.]

Figure 6 reports the impulse responses of investment rate to credit spread and asset volatility using the two specifications. Credit spread has a negative impact on investment while asset volatility has a positive impact. As shown in panel (a) and (c) of Figure 6, the positive impact of asset volatility on investment is economically and statistically larger in the first specification, highlighting the importance of controlling for credit spread for asset volatility to be a strong positive signal for investment in the aggregate.

Market Volatility Eisdorfer (2008) also explores the different impacts of uncertainty on firm investment in the cross-section and finds that uncertainty has a positive effect on distressed firms' investment, seemingly opposite to our results. He uses expected market volatility which is generated by applying a GARCH (1,1) model to monthly returns of the NYSE market index. This poses the question whether our results would be different using aggregate volatility instead of firm-level idiosyncratic volatility.

[Table 13 and Table 14 here.]

To address this question, we first replicate Eisdorfer's (2008) results in Table 13. Columns 1-2 presents the results under Eisdorfer's (2008) specification, where we split the sample into financially healthy and distressed firms[11] and regress investment

[10] We use the value-weighted average of $\sigma_{i,t}$, $s_{i,t}$ and $[I/K]_{i,t}$ to generate the corresponding aggregate time series. We seasonally adjust the investment rate time series by using its four-quarter moving average. All variables are detrended using the HP filter with weight 1600.

[11] We select the 20th percentile of distance-to-default as the cutoff for distressed firms. Eisdorfer (2008) classifies firms with Z-score below 1.81 at the beginning of each year as distressed, which generates a subsample of distressed firms including 18.6% of total observations.

rate on aggregate equity volatility, along with the same control variables.[12] The coefficient on aggregate equity volatility is significantly negative for healthy firms and is positive but insignificant for firms closer to default. Column 3-5 presents the result under our specification, where we sort firms into tercile groups based on credit spread each quarter and regress investment rate on aggregate equity volatility in addition to idiosyncratic equity volatility and credit spread. The coefficient on idiosyncratic equity volatility still goes from significantly positive to significantly negative as firm's credit spread goes up, while the coefficient on aggregate equity volatility goes in the opposite direction. We emphasize that equity volatility is an ambiguous signal for investment, driven by asset volatility and leverage, so we run the regressions using asset volatility. As shown in Table 14, both the coefficients on aggregate asset volatility and firm-level idiosyncratic asset volatility are positive across all subgroups when controlling for firm-level credit spread, or aggregate credit spread, or both.

5 Investment Decisions with Debt Overhang

In this section, we develop a simple but general credit risk model to analyze the investment choices of a firm with outstanding debt already in place. Two forces drive the investment decision: debt overhang and the option value of equity. We demonstrate that credit spreads and asset volatility are jointly unambiguous signals of these two forces. However, the signals provided by leverage and asset volatility or credit spreads and equity volatility are ambiguous and can change in the cross-section. All proofs are relegated to Appendix D. For ease of notation, we sometimes write $f_x(x) \equiv \frac{\partial f(x)}{\partial x}$.

Consider a firm that has risky assets in place and has funded itself partly with debt. In the first period, shareholders choose how much to invest. At the beginning

[12]The control variables are: firm size, estimated by the log the market value of the firm's total assets; market-to-book ratio; leverage, estimated by the ratio of the book value of total debt to the book value of total assets; cash flow, estimated by the ratio of operating cash flow to PP&E at the beginning of the year; the NBER recession dummy variable; the BAA-AAA yield spread; the interest rate, estimated by the nominal return on 1-month Treasury bills.

of the second period, a random productivity shock is realized, and, after observing the payoff of their investment, shareholders decide whether to file for bankruptcy or not. For our basic argument, we make the following assumptions regarding the firm and its investments.

Assumption 5.1 (Assets in Place)**.** *The firm has existing real assets in place with a final value of $Y(\iota, z)$, which is a function of investment ι and a random productivity shock z realized in the future. The assets are normalized to have an initial value of one. That is, $\mathbb{E}[Y(0, z)] = 1$.*

Assumption 5.2 (Firm Liabilities)**.** *The firm is funded by equity, together with a debt claim with total face value b that is due in the second period when the asset returns are realized. In the second period, shareholders decide whether to default. Upon bankruptcy, the entirety of the firm's value is lost. Furthermore, shareholders cannot liquidate the firm ($\iota \geq 0$).*

We show that our results are robust to a relaxation of Assumption 5.2 featuring partial recovery in Section D.

Assumption 5.3 (Pricing)**.** *All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities equal to their expected payoff with respect to a risk-neutral distribution $F(z; \sigma)$ of firm's asset productivity z, and $Y(\iota, z)$ with full support on $[0, \infty)$.*

Given our assumptions about payouts and pricing, it follows that the value of equity E and debt D are given by:

$$E(b, \iota, \underline{z}, \sigma) = \int_{\underline{z}}^{\infty} (Y(\iota, z) - b) dF(z; \sigma) - \iota,$$
$$D(b, \underline{z}, \sigma) = (1 - F(\underline{z}; \sigma))b.$$

The first order conditions for investment ι and the default threshold $\underline{z}$ imply that, at an optimum, ι and $\underline{z}$ satisfy:

$$\int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) dF(z;\sigma) = 1,$$
$$Y(\iota, \underline{z}) = b.$$

Credit spreads are given by: $cs(\underline{z}, \sigma) = F(\underline{z}; \sigma)$. To streamline our analysis, we also make assumptions on the risk distribution.

Assumption 5.4 (Investment Returns)**.** *The investment return function $Y(\iota, z)$ is continuous in both ι and z, positive, homogeneous of degree 1 in z, strictly increasing in ι, strictly concave in ι, and normalized to z when $\iota = 0$. That is,*

$$Y(\iota, z) = k(\iota)z \geq 0, \qquad k(0) = 1, \qquad k_{\iota}(\iota) > 0, \qquad k_{\iota\iota}(\iota) < 0.$$

Furthermore, the standard deviation σ of z is a finite moment of the distribution F.

Assumption 5.4 imposes restrictions common in models with investment. The homogeneity of degree 1 in z allows us to disentangle the effect of investment i and risk z on the investment returns $Y(\iota, z)$ and simplifies the analytics. Assumption 5.5 provides the only assumptions we make on the distribution of productivity shocks, $F(z)$. These assumptions are always satisfied with the Black–Scholes–Merton model and most risk distributions usually considered in finance.

Assumption 5.5 (Vega)**.** *The distribution of the productivity shock $F(z;\sigma)$ is such that vega is always positive:*

$$\nu(\underline{z}, \sigma) = \frac{\partial}{\partial \sigma} \mathbb{E}\left[(z - \underline{z})^{+}\right] > 0.$$

The model has two free parameters, leverage and asset volatility. Given the normalization $\mathbb{E}\left[Y(0, z)\right] = 1$, at the beginning of the first period leverage is simply the face value of debt b with zero investment. The model has two endogenous decision variables, investment ι and the default threshold $\underline{z}$. We use this simple model to study the behavior of investment following changes in the key observable variables from our empirical section: asset volatility σ, leverage b, credit spreads cs, and equity volatility σ^e.

Proposition 5.6 (Credit Spread and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to credit spread is given by:*

$$\frac{\partial \iota}{\partial cs} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} < 0, \tag{2}$$

where $\mu(\underline{z}, \sigma) = \mathbb{E}\left[z | z \geq \underline{z}\right] \mathbb{P}[z \geq \underline{z}]$. *Holding credit spread constant, the partial derivative of investment with respect to volatility is given by:*

$$\frac{\partial \iota}{\partial \sigma} = -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)} > 0. \tag{3}$$

In Proposition 5.6, we provide the elasticities of investment when observing asset volatility and credit spread. Given Assumptions 5.1-5.5, the sign of these partial derivatives match our empirical results. The first term on the right-hand side of equation (2) is negative due to the concavity of $k(\iota)$ in the denominator. All other terms are positive and thus the sign of the elasticity of investment to credit spreads is always negative.

In terms of the magnitude of the negative effect of higher credit spreads on investment, consider the denominator of the second term on the right-hand side of equation (2), $\mu(\underline{z}, \sigma)$. This term represents the expected return on one unit of investment given the option to default. As the default boundary $\underline{z}$ (which is also the marginal product lost from increasing the default threshold) increases, expected returns $\mu(\underline{z}, \sigma)$ decrease and shareholders have less incentives to invest. Thus, the debt-overhang problem intensifies when the firm gets closer to default. We also note the role of the concavity of the investment return function. If effective capital is more concave in investment, the first term will be smaller because firms won't have to adjust investment as much since $k_\iota(\iota)$ increases faster for a given reduction in investment.

By contrast, investment reacts positively to an increase in volatility as the payout to shareholders is non-linear with limited downside and unlimited upside, that is,

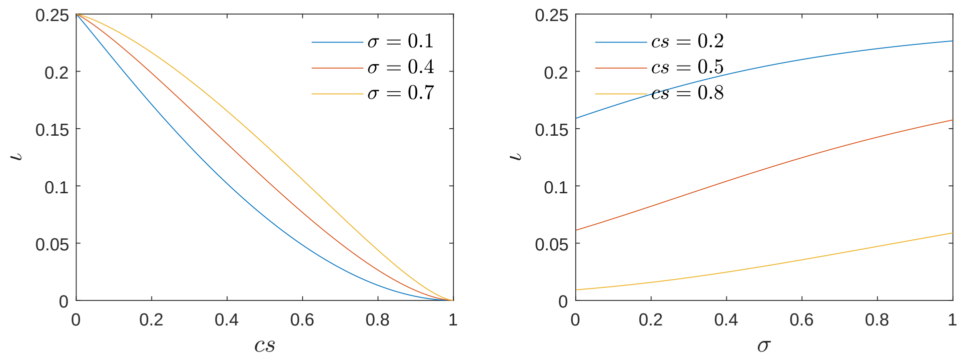


Figure 1: Optimal investment with log-normal distribution. The left picture shows the level of investment ι as a function of credit spreads cs for different levels of asset volatility σ, while the right figure shows the level of investment ι as a function of asset volatility σ for different levels of credit spreads cs. The production function is given by: $Y(\iota, z) = z(1 + \iota^{\alpha})$ where $\alpha = 0.5$.

vega $\nu(\underline{z}, \sigma)$ is positive. In equation (3), the first term is the same, except for the negative sign in front of it, while the numerator of the second term reflects the option value of higher investment as volatility increases. How strong the option-value effect is depends on the distribution of productivity shocks, $F(z; \sigma)$. Again, the ratio of the marginal investment return $k_{\iota}(\iota)$ to investment return concavity $k_{\iota\iota}(\iota)$ determines the strength of the investment response. If the marginal investment return $k_{\iota}(\iota)$ is large or the marginal productivity does not fall too fast (low $|k_{\iota\iota}(\iota)|$), then the investment response to a change in volatility is stronger.

Thus, in this simple model with fairly general as well as fairly standard assumptions, the signs of the effects of credit spreads and asset volatility on investment are unambiguous. Changes in credit spreads cs signal changes in the debt-overhang burden and changes in asset volatility σ signal changes in the option value of equity. In Figure 1, we illustrate the optimal investment function with a log-normal distribution of risk.

We now compare the straightforward roles of credit spreads and asset volatility in determining investment with the more intricate relation between *leverage* and asset volatility in investment decisions. This analysis exemplifies why credit spreads and asset volatility are clean empirical measures of the effects of financial soundness and option value on investment decisions.

Proposition 5.7 (Leverage and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to leverage is given by:*

$$\frac{\partial \iota}{\partial b} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} \xi_{b|\sigma}(\iota, \underline{z}, \sigma) < 0, \tag{4}$$

where

$$\xi_{b|\sigma}(\iota, \underline{z}, \sigma) \equiv \frac{f(\underline{z}; \sigma)}{k(\iota)} \varphi(\underline{z}, \sigma) > 0, \qquad \varphi(\iota, \underline{z}, \sigma) \equiv \left(1 + \frac{k_\iota(\iota)^2 \underline{z}^2 f(\underline{z}; \sigma)}{k(\iota) k_{\iota\iota}(\iota) \mu(\underline{z}; \sigma)}\right)^{-1} > 0.$$

Holding leverage constant, the partial derivative of investment with respect to volatility is given by:

$$\frac{\partial \iota}{\partial \sigma} = -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}; \sigma)} \xi_{\sigma|b}(\iota, \underline{z}, \sigma), \tag{5}$$

where

$$\xi_{\sigma|b}(\iota, \underline{z}, \sigma) \equiv \left(1 - \frac{\underline{z} F_\sigma(\underline{z}; \sigma)}{\nu(\underline{z}, \sigma)}\right) \varphi(\iota, \underline{z}, \sigma).$$

Proposition 5.7 shows that if, instead of controlling for credit spreads cs, we observe leverage b, the elasticities of investment become more intricate. Starting with equation (4), note that the first two terms on the right-hand side are equivalent to those in Proposition 5.6. The wedge $\xi_{b|\sigma}$ captures the additional effects of changing leverage. Note that φ is always positive at a maxima as imposed by the second-order conditions. Thus, this wedge is always positive, and the sign of the effect of leverage on investment holding asset volatility constant is always negative. The φ term captures the feedback loop between investment and default decisions. Following a decrease in investment, shareholders default more often as output decreases and thus incentives to pay back the debt also decrease. That additional force was not present in Proposition 5.6, since changing credit spreads $F(\underline{z}; \sigma)$ controls for the default decision $\underline{z}$ directly. Holding leverage constant instead controls for $b = Y(\iota, \underline{z})$, which is a function of both ι and $\underline{z}$.

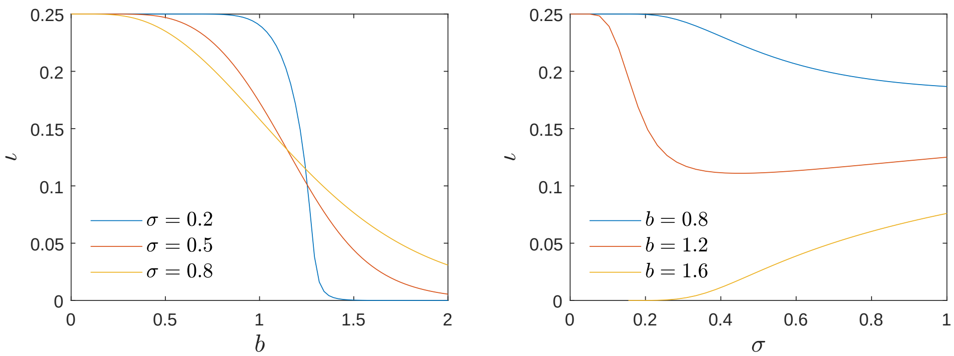


Figure 2: Optimal investment with log-normal distribution. The left picture shows the level of investment ι as a function of leverage b for different levels of asset volatility σ, while the right figure shows the level of investment ι as a function of asset volatility σ for different levels of leverage b. The production function is given by: $Y(\iota, z) = z(1 + \iota^{\alpha})$ where $\alpha = 0.5$.

Turning to the effect of asset volatility on investment holding leverage constant, the sign now becomes ambiguous. Relative to the effect holding credit spread constant characterized in Proposition 5.6, there is again a wedge, which we denote $\xi_{\sigma|b}$. Intuitively, there are two effects to increasing asset volatility holding leverage constant. The first is that the option value of investment increases. The second is that the debt overhang problem also increases. To hold leverage $b = Y(\iota, \underline{z})$ constant as asset volatility increases, the default threshold $\underline{z}$ must change and the distance to default could shrink faster than the increase in the option value. The wedge $\xi_{\sigma|b}$ captures this horserace between option value and what is lost in default as asset volatility increases. If the option value effect is strong, this term will be positive. However, if the increase in asset volatility moves a large probability mass into the default region (thus $\underline{z}F_{\sigma}(\underline{z};\sigma)$ is positive), the term can be negative. In other words, when the marginal increase in investment returns lost to default $zF_{\sigma}(\underline{z};\sigma)$ dominates the marginal increase in the option value $\nu(\underline{z},\sigma)$, shareholders reduce investment following an increase in volatility.

Which effect dominates is highly dependent on the shape of the distribution $F(z;\sigma)$. In Figure 2, we plot the optimal investment decision as a function of asset volatility σ when holding leverage b constant assuming a log-normal distributions for z. The monotonic relation between leverage and investment holding asset volatility constant is clear. However, the relation between investment and asset volatility holding leverage constant is non-monotonic. When leverage is high, the option-value effect dominates while the debt-overhang effect dominates when leverage is low.

Next, we consider the changes in investment when observing credit spreads and equity volatility, and illustrate the intuition our model suggests for the empirical finding that the sign of the elasticity of investment with respect to equity volatility changes sign in the cross section of more and less distressed firms. First, we define equity volatility as measured in the data as:[13]

$$\sigma^e(\underline{z}, \sigma) = \frac{\sigma}{E(b, 0, \underline{z}, \sigma)} = \frac{\sigma}{\mu(\underline{z}, \sigma)},$$

where $\mu(\underline{z}, \sigma) = \mathbb{E}\left[(z - \underline{z})^{+}\right]$ is the unconditional left-truncated expectation of the payoff above the default threshold. Thus, equity is levered asset volatility, where the denominator $\mu(\underline{z}, \sigma)$ represents the impact of leverage on equity volatility. If the debt burden from leverage b increases, then the default threshold $\underline{z}$ increases as well and equity's expected payoff $\mu(\underline{z}, \sigma)$ decreases. Conversely, if the firm is funded entirely by equity ($b = 0$), then $\underline{z}$ is equal to zero—the lower bound of the support. In that case, equity volatility is equal to asset volatility ($\sigma^e(\underline{z}, \sigma) = \sigma$) since $\mu(0, \sigma) = 1$.

Proposition 5.8 (Credit Spread and Equity Volatility)**.** *Holding equity volatility constant, the partial derivative of investment with respect to credit spreads is given by:*

$$\frac{\partial \iota}{\partial cs} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} \xi_{cs|\sigma^e}(\underline{z}, \sigma), \tag{6}$$

[13] Defining equity volatility as:

$$\sigma^e(\underline{z}, \sigma) = \frac{\sqrt{\mathbb{V}ar\left[(Y(\iota, z) - b)\,\mathbb{1}\{z \geq \underline{z}\} - \iota\right]}}{E(b, 0, \underline{z}, \sigma)} = \frac{\sigma(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)},$$

where $\sigma(\underline{z}, \sigma) = \sqrt{\mathbb{V}ar\left[(z - \underline{z})^{+}\right]}$ makes the analysis untractable and is further away from how equity volatility is measured as the truncation of the volatility is not reflected in the measurement of equity volatility unless default occurred.

where

$$\xi_{cs|\sigma^e}(\underline{z},\sigma) \equiv 1 + \sigma^e_{\underline{z}}(\underline{z},\sigma)\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z})}\xi_{\sigma^e|cs}(\underline{z},\sigma).$$

Holding credit spread constant, the partial derivative of investment with respect to equity volatility is given by:

$$\frac{\partial \iota}{\partial \sigma^e} = -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\nu(\underline{z})}{\mu(\underline{z},\sigma)}\xi_{\sigma^e|cs}(\underline{z},\sigma), \tag{7}$$

where

$$\xi_{\sigma^e|cs}(\underline{z},\sigma) \equiv \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.$$

We again use wedges to make the distinction between Propositions 5.8 and 5.6 clear. It is easiest to start with the relation between investment and equity volatility holding credit spread constant. To understand the additional complication when using equity volatility as a signal of uncertainty, it is useful to look at the partial derivative of equity volatility with respect to asset volatility σ and the default threshold $\underline{z}$:

$$\sigma^e_\sigma(\underline{z},\sigma) = \frac{1}{\mu(\underline{z},\sigma)} - \frac{\sigma\nu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)^2} \qquad \text{and} \qquad \sigma^e_{\underline{z}}(\underline{z},\sigma) = \frac{\sigma\left(1 - F(\underline{z};\sigma)\right)}{\mu(\underline{z},\sigma)^2} \geq 0.$$

Thus, when the option value impact of asset volatility $\nu(\underline{z},\sigma)$ is large, equity volatility decreases following a positive shock to asset volatility. Indeed, the increase in the payoff to equity holders (denominator of σ^e) gets larger than the relative increase in asset volatility (numerator of σ^e). Add to that effect that to keep the credit spread cs constant, the default threshold $\underline{z}$ needs to decrease, and it not surprising anymore that following a positive asset volatility shock, equity volatility might decrease. Corollary 5.9 makes that argument explicit.

Corollary 5.9 (Equity Volatility and Asset Volatility)**.** *If the total derivative of the* *default threshold with respect to asset volatility is such that:*

$$\frac{d\underline{z}}{d\sigma} < \frac{\sigma\nu(\underline{z},\sigma) - \mu(\underline{z},\sigma)}{\sigma(1 - F(\underline{z};\sigma))},$$

then the total derivative of equity volatility with respect to asset volatility is negative:

$$\frac{d\sigma^e(\underline{z},\sigma)}{d\sigma} < 0.$$

These additional forces are captured by the wedges $\xi_{\sigma^e|cs}(\underline{z},\sigma)$ and $\xi_{\sigma^e|cs}(\underline{z},\sigma)$ such that the signs of the elasticities of Proposition 5.8 are highly dependent on the shape of the risk distribution $F(z;\sigma)$ and the level of leverage and volatility of the firm, contrarily to the robust signs of the elasticities of Proposition 5.6.

Lemma 5.10 (Existence of Credit Spread and Equity Volatility Pair)**.** *Given* $(cs, \sigma^e) \in [0,1] \times \mathbb{R}^+$*, there does not always exist a solution* $(\underline{z},\sigma) \in \mathbb{R}^+ \times \mathbb{R}^+$ *to the following system of two equations:*

$$cs = F(\underline{z};\sigma),$$
$$\sigma^e = \frac{\sigma}{\mu(\underline{z},\sigma)}.$$

Furthermore, the solution might not be unique.

Following Lemma 5.10, these non-monotonicities also complicate the mapping of investment decisions in the (cs,σ^e)-space. Thus, in Figure 3, we show the sign of the wedges in the (cs,σ)-space for two distributions: a log-normal distribution and a log-normal mixture distribution. In the case of the log-normal distribution, the wedges are either both positive (white area), such that the signs of the elasticities are identical to Proposition 5.6, or both negative (light gray area), such that the signs of the elasticities are opposite to Proposition 5.6.

The mixture distribution is a mixture of two log-normal distributions (see caption of Figure 3) and therefore bimodal. This risk distribution could correspond to a technology where the productivity shock is drawn from either a bad (low mean) or a

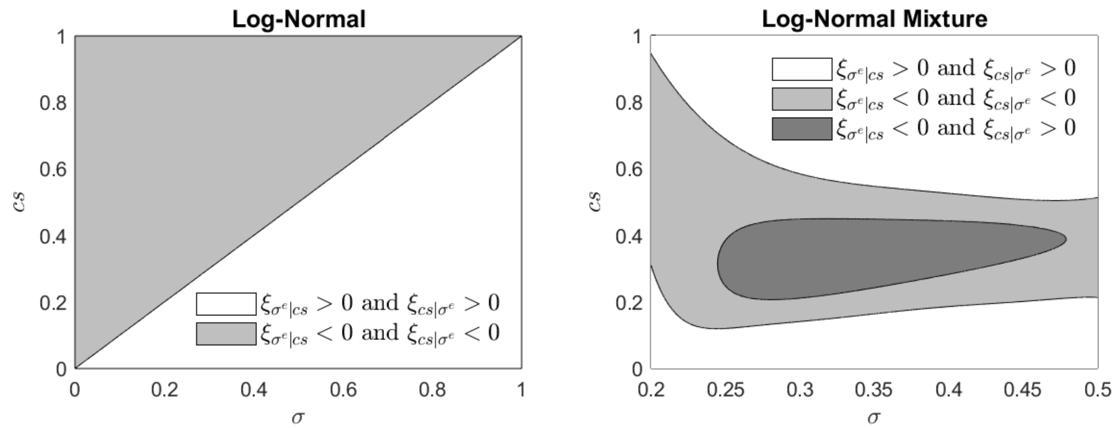


Figure 3: Sign of Wedges for Log-Normal and Log-Normal Mixture. These pictures shows the sign of the wedges of Proposition 5.8 in the (cs,σ)-space for the log-normal distribution (left) and a log-normal mixture distribution (right). The mixture distribution is a mixture of two log-normal distributions drawn with 50% probability with parameters $(\mu_1, \hat{\sigma})$ and $(\mu_2, \hat{\sigma})$ such that the unconditional mean of z is 1 and the standard deviation of z is σ. We set $\hat{\sigma} = 0.2$ in this example.

good (high mean) distribution. In this case, an increase in uncertainty could have a large effect on the option value without substantially impacting default risk. Thus, a third area (dark grey) appears, where the elasticities with respect to credit spread and equity volatility are both negative. In the example of Figure 3, fixing asset volatility to 0.3, the elasticity with respect to equity volatility is positive for low credit spread level ($cs \leq 0.15$) and gets negative for high level of leverage ($0.15 \leq cs \leq 0.6$). At the same time, the elasticity with respect to credit spread is mostly negative (for $cs \leq 0.14$ and $0.21 \leq cs \leq 0.45$). Thus, in that example, we observe the same change of sign in the cross-section as in our empirical results.

In Appendix E, we show that our results hold in a setting with endogenous leverage dynamics. We extend the framework of DeMarzo and He (2020) to include an investment function and show that Proposition 5.6 still holds.

6 Conclusion

In our empirical analysis and model, we establish that equity volatility is an ambiguous signal of uncertainty for firm-level investment decisions. Intuitively, if a positive uncertainty shock causes a large increase in the option value of equity, equity volatility might go down. Using asset volatility instead results in an unambiguous relationship with investment: an increase in asset volatility generates an increase in the investment rate.

Overall, our model and evidence provide support for the idea that the close connection between bond markets and the macroeconomy is due to the unique non-linear transformation of asset volatility and leverage that credit spreads represent.

References

Alfaro, Ivan, Bloom, Nicholas, and Lin, Xiaoji. The finance uncertainty multiplier. Technical report, National Bureau of Economic Research, 2018.

Arellano, Cristina, Bai, Yan, and Kehoe, Patrick J. Financial frictions and fluctuations in volatility. *Journal of Political Economy*, 127(5):2049–2103, 2019.

Arora, Navneet, Bohn, Jeffrey R, and Zhu, Fanlin. Reduced form vs. structural models of credit risk: A case study of three models. *Journal of Investment Management*, 3(4):43, 2005.

Atkeson, Andrew G, Eisfeldt, Andrea L, and Weill, Pierre-Olivier. Measuring the financial soundness of us firms, 1926–2012. *Research in Economics*, 71(3):613–635, 2017.

Bernanke, Ben S. On the predictive power of interest rates and interest rate spreads, 1990.

Bharath, Sreedhar T and Shumway, Tyler. Forecasting default with the merton distance to default model. *The Review of Financial Studies*, 21(3):1339–1369, 2008.

Black, Fischer and Scholes, Myron. The pricing of options and corporate liabilities. *The Journal of Political Economy*, 81(3):637–654, 1973.

Bloom, Nicholas. The impact of uncertainty shocks. *econometrica*, 77(3):623–685, 2009.

Carhart, Mark M. On persistence in mutual fund performance. *The Journal of finance*, 52(1):57–82, 1997.

Christiano, Lawrence J, Motto, Roberto, and Rostagno, Massimo. Risk shocks. *American Economic Review*, 104(1):27–65, 2014.

Collin-Dufresn, Pierre, Goldstein, Robert S, and Martin, J Spencer. The determinants of credit spread changes. *The Journal of Finance*, 56(6):2177–2207, 2001.

Davies, J, Fama, EF, and French, KR. Characteristics, covariances, and average returns: 1927–1997. *Journal of Finance*, 55:389–406, 2000.

DeMarzo, Peter M and He, Zhiguo. Leverage dynamics without commitment. *The Journal of Finance*, 2020.

Eisdorfer, Assaf. Empirical evidence of risk shifting in financially distressed firms. *The Journal of Finance*, 63(2):609–637, 2008.

Erickson, Timothy and Whited, Toni M. Treating measurement error in tobin's q. *The Review of Financial Studies*, 25(4):1286–1329, 2012.

Friedman, Benjamin M and Kuttner, Kenneth N. Money, income, prices, and interest rates. *The American Economic Review*, pages 472–492, 1992.

Friedman, Benjamin M and Kuttner, Kenneth N. Indicator properties of the paper—bill spread: Lessons from recent experience. *Review of Economics and Statistics*, 80(1):34–44, 1998.

Gertler, Mark and Lown, Cara S. The information in the high-yield bond spread for the business cycle: evidence and some implications. *Oxford Review of economic policy*, 15(3):132–150, 1999.

Giesecke, Kay, Longstaff, Francis A, Schaefer, Stephen, and Strebulaev, Ilya A. Macroeconomic effects of corporate default crisis: A long-term perspective. *Journal of Financial Economics*, 111(2):297–310, 2014.

Gilchrist, Simon and Zakrajšek, Egon. Credit spreads and business cycle fluctuations. *American economic review*, 102(4):1692–1720, 2012.

Gilchrist, Simon, Yankov, Vladimir, and Zakrajšek, Egon. Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets. *Journal of monetary Economics*, 56(4):471–493, 2009.

Gilchrist, Simon, Sim, Jae W, and Zakrajšek, Egon. Uncertainty, financial frictions, and investment dynamics. Technical report, National Bureau of Economic Research, 2014.

Gürkaynak, Refet S, Sack, Brian, and Wright, Jonathan H. The us treasury yield curve: 1961 to the present. *Journal of monetary Economics*, 54(8):2291–2304, 2007.

Kermani, Amir and Ma, Yueran. Two tales of debt. Technical report, National Bureau of Economic Research, 2020.

Krishnamurthy, Arvind and Muir, Tyler. How credit cycles across a financial crisis. Technical report, National Bureau of Economic Research, 2017.

Leland, Hayne E. Corporate debt value, bond covenants, and optimal capital structure. *The journal of finance*, 49(4):1213–1252, 1994.

Merton, Robert C. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of finance*, 29(2):449–470, 1974.

Nazeran, Pooya and Dwyer, Douglas. Credit risk modeling of public firms: Edf9. *Moody's Analytics White Paper*, 2015.

Panousi, Vasia and Papanikolaou, Dimitris. Investment, idiosyncratic risk, and ownership. *The Journal of finance*, 67(3):1113–1148, 2012.

Philippon, Thomas. The bond market's q. *The Quarterly Journal of Economics*, 124 (3):1011–1056, 2009.

Appendices

A Figures

Figure 4: This figure presents the elasticity of investment with respect to equity volatility across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log credit spread: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 2 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_1 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

.

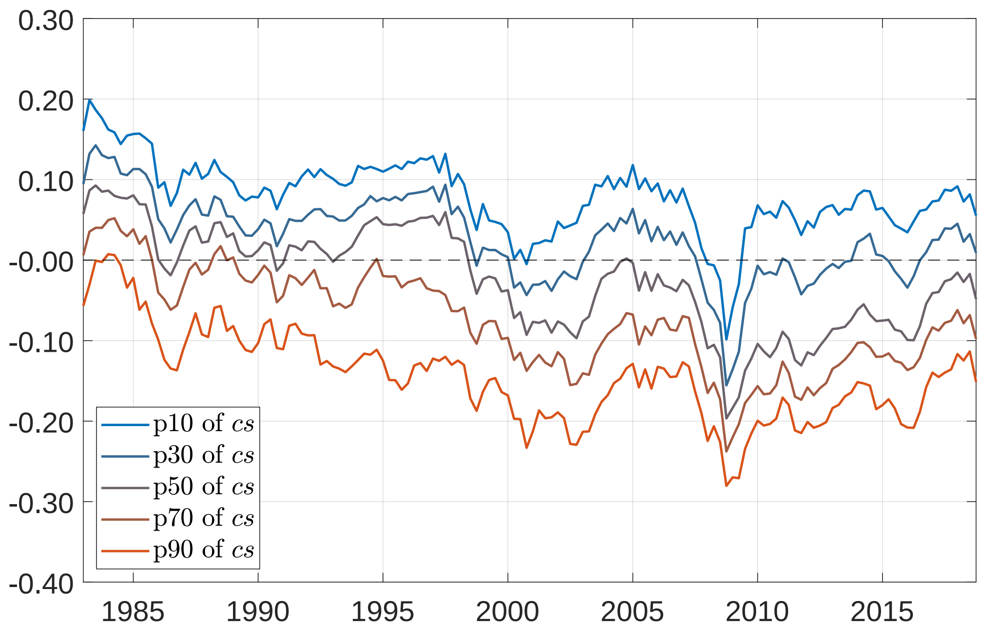


Figure 5: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log equity volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 2 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

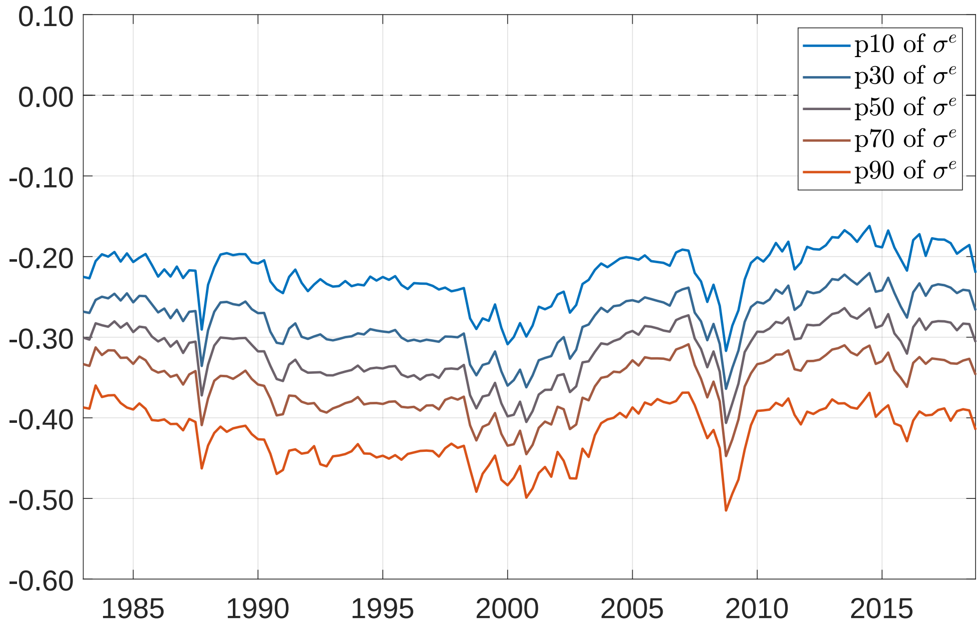
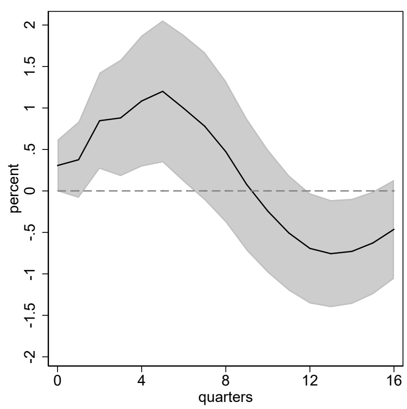
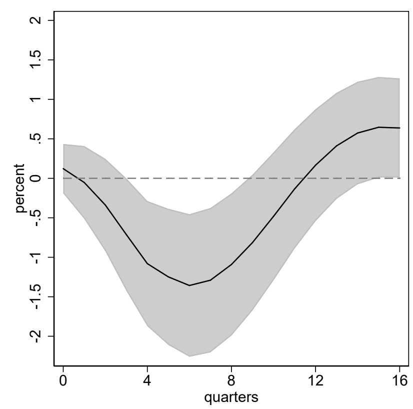


Figure 6: This figure plots the impulse responses of investment to an orthogonalized 1 standard deviation shock to asset volatility and credit spread. The VAR is estimated using four lags of each endogenous variable. Subfigures (a) and (b) correspond to the recursive ordering: $(cs,\ \sigma,\ I/K)$. Subfigures (c) and (d) correspond to the recursive ordering: $(\sigma,\ cs,\ I/K)$. The shaded bands represent the 95% confidence interval.

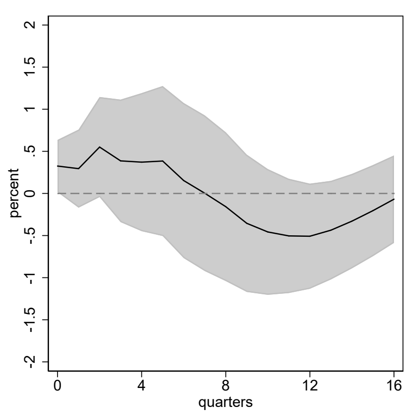
(a) Asset volatility



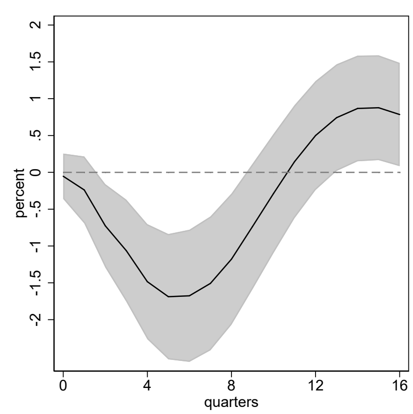
(b) Credit spread



(c) Asset volatility



(d) Credit spread



B Tables

Table 1: This table documents the summary statistics of our firm-quarter variables.

	count	mean	sd	min	max
idiosyncratic equity volatility	42580	0.28	0.15	0.09	1.13
idiosyncratic asset volatility	38421	0.19	0.09	0.07	0.78
implied equity volatility	18340	0.34	0.14	0.14	0.96
implied asset volatility	18183	0.18	0.07	0.05	0.45
market leverage	41924	2.30	1.63	1.12	17.25
credit spreads	42580	296.11	230.55	50.38	1485.43
fair value spreads	30906	146.77	232.74	9.15	1530.69
roa	39682	0.04	0.02	-0.03	0.10
tangibility ratio	33800	0.71	0.41	0.04	2.19
sales ratio	41865	1.39	2.42	0.05	25.16
income ratio	39784	0.19	0.26	-0.11	2.46
Tobin's	31944	2.36	3.12	-0.22	23.97

Table 2: This table documents the relationship between investment, equity volatility, and credit spread at the firm-quarter level. Sample period is from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.152***		-0.057***	0.046**	-0.041*	-0.095***	0.660***	0.477***
	(-8.02)		(-3.63)	(2.27)	(-1.85)	(-3.50)	(7.14)	(4.63)
$\log cs_{i,t-1}$		-0.270***	-0.253***	-0.097**	-0.225***	-0.393***	-0.423***	-0.272***
		(-12.45)	(-12.23)	(-2.40)	(-4.94)	(-8.78)	(-14.05)	(-8.07)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$							-0.129***	-0.089***
							(-7.64)	(-4.69)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	42580	42580	42580	13548	13583	12905	42580	28781
R-squared	0.111	0.134	0.135	0.161	0.137	0.115	0.141	0.224

Table 3: This table documents the relationship between investment, equity volatility, and fair value spread at the firm-quarter level. Sample period is from 1983 to 2018. Sample period is from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.140***		0.005	0.059***	0.015	-0.046	0.265***	0.200***
	(-6.75)		(0.30)	(3.10)	(0.64)	(-1.59)	(5.73)	(4.17)
$\log \widehat{cs}_{i,t-1}$		-0.150***	-0.151***	-0.090***	-0.097***	-0.194***	-0.227***	-0.141***
		(-13.19)	(-13.63)	(-3.16)	(-3.93)	(-9.65)	(-13.00)	(-7.47)
$\log \sigma^e_{i,t-1} \times \log \widehat{cs}_{i,t-1}$							-0.060***	-0.044***
							(-5.72)	(-3.81)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	30808	30808	30808	9798	9850	9606	30808	20787
R-squared	0.115	0.152	0.152	0.180	0.146	0.134	0.156	0.221

Table 4: This table documents the relationship between investment, asset volatility, and credit spread at the firm-quarter level. Sample period is from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma_{i,t-1}$	0.060***		0.087***	0.096***	0.059***	0.122***	0.064***
	(3.80)		(5.84)	(4.81)	(2.74)	(4.99)	(4.03)
$\log cs_{i,t-1}$		-0.262***	-0.269***	-0.075*	-0.265***	-0.442***	-0.149***
		(-11.68)	(-12.04)	(-1.83)	(-5.58)	(-8.90)	(-6.27)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	38833	38833	38833	12612	12371	11707	26332
R-squared	0.107	0.135	0.137	0.171	0.139	0.117	0.222

Table 5: This table documents the relationship between investment, implied asset volatility, and credit spreads at the firm-quarter level. Sample period is from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all
$\log \widehat{\sigma}_{i,t-1}$	0.275***		0.250***	0.233***	0.249***	0.232***	0.187***
	(8.18)		(7.69)	(5.25)	(4.36)	(4.17)	(5.27)
$\log cs_{i,t-1}$		-0.328***	-0.317***	-0.120***	-0.286***	-0.625***	-0.177***
		(-9.15)	(-8.77)	(-2.65)	(-4.28)	(-6.10)	(-5.13)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	18441	18441	18441	7376	6089	4095	12808
R-squared	0.128	0.151	0.164	0.180	0.148	0.154	0.241

Table 6: This table documents the relationship between investment, implied asset volatility, market leverage, and credit spreads at the firm-quarter level. Sample period is from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \widehat{\sigma}_{i,t-1}$	0.275***		-0.125***	0.042	-0.045	-0.218***	0.959***	0.669***
	(8.18)		(-2.95)	(0.79)	(-0.66)	(-3.07)	(5.11)	(3.41)
$\log[MA/ME]_{i,t-1}$		-0.618***	-0.695***	-0.446***	-0.651***	-0.710***	-0.635***	-0.564***
		(-14.35)	(-12.65)	(-4.97)	(-7.49)	(-8.76)	(-11.96)	(-7.87)
$\log cs_{i,t-1}$							-0.448***	-0.286***
							(-6.72)	(-4.13)
$\log \widehat{\sigma}_{i,t-1} \times \log cs_{i,t-1}$							-0.190***	-0.128***
							(-5.68)	(-3.70)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	18441	18211	18211	7171	6073	4091	18211	12653
R-squared	0.128	0.186	0.188	0.189	0.170	0.182	0.199	0.261

Table 7: This table presents the loadings of equity volatility and credit spread on asset volatility and market leverage at the firm-quarter level. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	$\log \sigma^e_{i,t}$	$\log cs_{i,t}$	Panel B: Changes	$\Delta \log \sigma^e_{i,t}$	$\Delta \log cs_{i,t}$
$\log \sigma_{i,t}$	0.793***	0.154***	$\Delta \log \sigma_{i,t}$	0.754***	0.017***
	(87.78)	(13.90)		(76.29)	(5.21)
$\log[MA/ME]_{i,t}$	0.452***	0.591***	$\Delta \log[MA/ME]_{i,t}$	0.228***	0.227***
	(42.20)	(26.70)		(20.70)	(21.07)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	40290	40021	Observations	39070	38746
R-squared	0.834	0.566	R-squared	0.736	0.336

Table 8: This table documents the relationship between investment, implied asset volatility, and credit spread at the firm-quarter level for different lags and leads. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.269***	-0.263***	-0.070***	-0.069***
	(-11.88)	(-11.47)	(-5.08)	(-4.93)
$\log \sigma_{i,t-1}$	0.083***		0.030***	
	(5.70)		(2.84)	
$\log \sigma_{i,t-2}$	0.013*		-0.005*	
	(1.87)		(-1.81)	
$\log \sigma_{i,t-3}$	0.010		0.015	
	(1.17)		(1.56)	
$\log \sigma_{i,t-4}$	0.015		0.004	
	(1.63)		(0.39)	
$\log \sigma_{i,t+1}$		0.016		0.005
		(1.62)		(1.09)
$\log \sigma_{i,t+2}$		0.012**		0.006**
		(2.11)		(2.32)
$\log \sigma_{i,t+3}$		0.006*		0.006
		(1.75)		(1.38)
$\log \sigma_{i,t+4}$		0.002		0.003
		(0.66)		(0.79)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	36702	33513	24789	22713
R-squared	0.140	0.140	0.392	0.391

Table 9: This table documents the relationship between investment, equity volatility, and credit spread at the firm-quarter level for different levels of research and development and investment ratios. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) low R&D	(2) high R&D	(3) low R&D	(4) high R&D	(5) low inv.	(6) high inv.	(7) low inv.	(8) high inv.
$\log \sigma^e_{i,t-1}$	0.942***	0.792***	0.353**	0.372**	0.528***	0.279***	0.326***	0.148*
	(4.07)	(3.63)	(2.25)	(2.31)	(4.82)	(3.48)	(3.35)	(1.72)
$\log cs_{i,t-1}$	-0.550***	-0.424***	-0.193***	-0.152***	-0.370***	-0.161***	-0.197***	-0.061**
	(-6.35)	(-6.35)	(-3.33)	(-2.61)	(-10.92)	(-6.52)	(-6.31)	(-2.16)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.170***	-0.155***	-0.063**	-0.073**	-0.104***	-0.061***	-0.065***	-0.031*
	(-4.18)	(-3.91)	(-2.26)	(-2.48)	(-5.34)	(-4.11)	(-3.72)	(-1.94)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls			✓	✓			✓	✓
Observations	4389	4429	3028	3421	13975	14064	9544	9758
R-squared	0.189	0.231	0.450	0.418	0.190	0.199	0.345	0.283

Table 10: This table documents the relationship between investment, level of implied asset volatility, shock to implied asset volatility, and credit spread at the firm-quarter level for different lags and leads. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$
$\log \widehat{\sigma}_{i,t-1}$	0.327***	0.297***	0.136***
	(8.81)	(8.19)	(5.66)
$\Delta \log \widehat{\sigma}_{i,t-1}$	-0.189***	-0.160***	-0.041*
	(-6.66)	(-5.75)	(-1.70)
$\log cs_{i,t-1}$		-0.311***	-0.085***
		(-8.86)	(-4.21)
Firm FE	✓	✓	✓
Time FE	✓	✓	✓
Controls			✓
Observations	18915	18915	12954
R-squared	0.131	0.166	0.391

Table 11: This table documents the instrumental variables regressions for equity and asset volatility. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) low cs	(2) mid cs	(3) high cs	(4) all
$\log \sigma^e_{i,t-1}$	0.828**	0.115	-0.237	
	(2.00)	(0.38)	(-0.64)	
$\log \sigma_{i,t-1}$				0.293*
				(1.74)
$\log cs_{i,t-1}$	-0.208***	-0.261***	-0.412**	-0.321***
	(-3.04)	(-3.31)	(-2.11)	(-11.21)
First Moments	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Observations	11943	12116	11507	34099
Underidentification	22.247	40.682	27.496	100.382
Weak identification	2.570	7.200	3.713	15.828

Table 12: This table documents the relationship between equity volatility, credit spread and investment for firms with different covenant tightness. Column 1 reports estimation results for the subsample with tight covenant (distance to threshold below median). Column 2 reports estimation results for the subsample with slack covenant (distance to threshold above median). Each observation is a firm-year. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

	(1) slack covenant	(2) tight covenant
$\log \sigma^e_{i,t-1}$	0.116	0.451**
	(0.66)	(2.23)
$\log cs_{i,t-1}$	-0.133**	-0.364***
	(-2.42)	(-5.19)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.019	-0.078**
	(-0.59)	(-2.26)
Firm FE	✓	✓
Time FE	✓	✓
Controls	✓	✓
Observations	8763	7525
R-squared	0.231	0.207

Table 13: This table documents the relationship between aggregate equity volatility and firm-level investment rate. Column 1 and column 2 report estimation of the regression model with aggregate equity volatility $\log \Sigma_t^e$ and control variables $X_{i,t}$ on the right-hand side for financially healthy and distressed firms. The control variables are: firm size, estimated by the log the market value of the firm's total assets; market-to-book ratio; leverage, estimated by the ratio of the book value of total debt to the book value of total assets; cash flow, estimated by the ratio of operating cash flow to PP&E at the beginning of the year; the NBER recession dummy variable; the BAA-AAA yield spread; the interest rate, estimated by the nominal return on 1-month Treasury bills. The financially healthy (distressed) firms are observations above (below) the 20th percentile of distance-to-default. Columns 3-5 report estimation of the regression model with aggregate equity volatility $\log \Sigma_t^e$, idiosyncratic firm-level equity volatility $\log \sigma_{i,t}^e$, and credit spread $\log s_{i,t}$ on the right-hand side use subsamples sorted by terciles based on credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \Sigma_{i,t}^e + \beta_2 \log \sigma_{i,t}^e + \beta_3 \log s_{i,t} + \gamma' \mathbf{X}_{i,t} + \eta_i + \epsilon_{i,t}$$

	(1) Healthy	(2) Distressed	(3) low cs	(4) mid cs	(5) high cs
$\log \Sigma_t^e$	-0.047***	0.033	-0.033***	0.045***	0.103***
	(-4.91)	(1.45)	(-3.12)	(3.47)	(6.27)
$\log \sigma_{i,t}^e$			0.145***	0.033	-0.096***
			(7.18)	(1.60)	(-4.14)
$\log s_{i,t}$			-0.084***	-0.239***	-0.364***
			(-3.38)	(-8.64)	(-12.70)
Controls	✓	✓			
Firm FE	✓	✓	✓	✓	✓
Observations	37831	7774	17614	17595	17205
R-squared	0.042	0.058	0.016	0.023	0.054

Table 14: This table documents the relationship between aggregate asset volatility, idiosyncratic asset volatility, and investment, controlling for firm-level or aggregate credit spread. Columns 1-3 control for firm-level credit spread. Columns 4-6 control for aggregate credit spread. Column 7-9 control for both, where we use $\log[s_{i,t}/S_t]$ for firm-level credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \Sigma_{i,t} + \beta_2 \log \sigma_{i,t} + \beta_3 \log[s_{i,t}/S_t] + \beta_4 \log S_t + \eta_i + \epsilon_{i,t}$$

	(1) low cs	(2) mid cs	(3) high cs	(4) low cs	(5) mid cs	(6) high cs	(7) low cs	(8) mid cs	(9) high cs
$\log \Sigma_t$	-0.020	0.083***	0.126***	0.057***	0.096***	0.080***	0.054***	0.092***	0.041
	(-1.23)	(4.14)	(4.86)	(3.23)	(4.60)	(2.73)	(3.03)	(4.39)	(1.42)
$\log \sigma_{i,t}$	0.162***	0.065***	0.051**	0.129***	0.058***	0.033	0.125***	0.064***	0.050**
	(7.95)	(3.31)	(2.35)	(6.89)	(2.94)	(1.50)	(6.52)	(3.26)	(2.33)
$\log s_{i,t}$	-0.086***	-0.263***	-0.405***						
	(-3.25)	(-8.69)	(-13.34)						
$\log S_t$				-0.211***	-0.316***	-0.272***	-0.205***	-0.283***	-0.229***
				(-5.91)	(-7.79)	(-5.13)	(-5.52)	(-7.02)	(-4.52)
$\log[s_{i,t}/S_t]$							0.026	-0.237***	-0.501***
							(0.88)	(-5.18)	(-12.25)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	16236	15947	15198	16236	15947	15198	16236	15947	15198
R-squared	0.021	0.026	0.049	0.030	0.019	0.008	0.030	0.026	0.053

C Robustness Checks

In this appendix, we provide several robustness checks for the results discussed above. In particular, in Table A1 - Table A4, we show that using lagged values of equity volatility, asset volatility and credit spread generates similar results, which highlights the predictive power of these measures on investment.

In table A5, we examine the loadings of credit spreads and equity volatility on asset volatility and leverage. Instead of using the firm's asset volatility and market leverage directly as in the main text, we use the average of all firms in the same industry (excluding itself) and generate similar results.

In Table A6 through Table A9, we present the regression results replicating those in Table 2 to Table 7 in the main text. The coefficients can be interpreted as the move in the dependent variable scaled by its standard deviation associated with one standard deviation increase in the explanatory variable. These results help us interpret the economic significance of the coefficients on equity volatility and credit spread. Also, the split-sample results confirm that our cross-sectional findings are not sensitive to different dispersion of the variables in different subgroups. In Table A10, we report results for regressions that estimate the same specification as in Table 2 while using the same sample as used to generate Table 3. Comparing Table A10 with Table 3 indicates that fair value spread behaves similarly to credit spread in our investment regressions, and there are no concerns over the sample selection since we are using exactly the same sample. In Figure A1, we show the elasticity of investment with respect to credit spread using asset volatility as the moderator variable and find very similar results those in Figure 5 in the main text.

Table A1: This table replicates Table 2 using lagged values. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and credit spread spread in columns 3-6, on credit spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \sigma^e_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t-1}$	-0.182***		-0.065***	0.038**	-0.052***	-0.105***	0.830***
	(-9.89)		(-4.44)	(2.02)	(-2.60)	(-4.54)	(9.85)
$\log s_{i,t-1}$		-0.293***	-0.272***	-0.115***	-0.286***	-0.513***	-0.477***
		(-13.44)	(-12.80)	(-3.70)	(-8.04)	(-12.46)	(-16.56)
$\log \sigma^e_{i,t-1} \times \log s_{i,t-1}$							-0.160***
							(-10.51)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	51228	51220	50820	16991	17007	16822	50820
R-squared	0.101	0.126	0.126	0.136	0.100	0.119	0.135

Table A2: This table replicate Table 3 using lagged values. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and fair value spread in columns 3-6, on fair value spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log \hat{s}_{i,t-1} + \gamma \log \sigma^e_{i,t-1} \times \log \hat{s}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t-1}$	-0.160***		-0.003	0.071***	0.024	-0.071***	0.334***
	(-8.29)		(-0.20)	(4.04)	(1.15)	(-2.72)	(7.52)
$\log \hat{s}_{i,t-1}$		-0.161***	-0.159***	-0.086***	-0.130***	-0.165***	-0.254***
		(-14.29)	(-14.34)	(-4.81)	(-8.93)	(-9.59)	(-14.82)
$\log \sigma^e_{i,t-1} \times \log \hat{s}_{i,t-1}$							-0.077***
							(-7.74)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	38740	37935	37600	12495	12534	12285	37600
R-squared	0.107	0.143	0.141	0.164	0.152	0.125	0.148

Table A3: This table replicates Table 4 using lagged values. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \sigma_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \sigma_{i,t-1}$	0.084***	0.086***	0.053***	0.062***	0.691***
	(5.79)	(5.14)	(2.76)	(2.78)	(7.20)
$\log s_{i,t-1}$	-0.284***	-0.104***	-0.305***	-0.512***	-0.466***
	(-12.61)	(-2.83)	(-6.56)	(-12.25)	(-13.14)
$\log \sigma_{i,t-1} \times \log s_{i,t-1}$					-0.110***
					(-6.37)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	46513	15869	15642	15002	46513
R-squared	0.126	0.147	0.124	0.117	0.130

Table A4: This table replicates Table 5 using lagged values. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \hat{\sigma}_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \hat{\sigma}_{i,t-1}$	0.272***	0.251***	0.251***	0.243***	0.825***
	(8.58)	(4.87)	(4.90)	(4.78)	(4.60)
$\log s_{i,t-1}$	-0.334***	-0.141***	-0.377***	-0.597***	-0.515***
	(-10.40)	(-3.23)	(-5.57)	(-8.03)	(-7.89)
$\log \hat{\sigma}_{i,t-1} \times \log s_{i,t-1}$					-0.098***
					(-3.09)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	22367	7526	7473	7316	22367
R-squared	0.154	0.149	0.139	0.166	0.156

Table A5: This table presents the loadings of credit spread on the industry average of asset volatility and market leverage. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. For a firm i in industry k at time t, we compute the industry average of log asset volatility excluding itself as $\frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$. We compute the industry average of market leverage similarly. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log y_{i,t} = \beta_1 \frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t} + \beta_2 \frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	(1) $\log \sigma^e_{i,t}$	(2) $\log s_{i,t}$	Panel B: Changes	(3) $\Delta \log \sigma^e_{i,t}$	(4) $\Delta \log s_{i,t}$
$\frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$	0.415***	0.055	$\Delta \frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$	0.292***	0.074***
	(11.21)	(0.87)		(11.71)	(4.58)
$\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.216***	0.353***	$\Delta \frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.135***	0.074***
	(5.38)	(5.30)		(3.70)	(3.15)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	47323	47246	Observations	44702	44636
R-squared	0.372	0.432	R-squared	0.163	0.329

Table A6: This table replicate Table 2 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log s_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.117***		-0.036***	0.049***	-0.011	-0.078***	0.563***
	(-8.99)		(-3.37)	(3.86)	(-0.73)	(-4.55)	(9.79)
$\log s_{i,t}$		-0.305***	-0.287***	-0.111***	-0.269***	-0.461***	-0.270***
		(-13.33)	(-12.84)	(-3.03)	(-5.68)	(-11.67)	(-12.40)
$\log \sigma^e_{i,t} \times \log s_{i,t}$							-0.107***
							(-10.21)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	52897	52900	52414	17614	17595	17205	52414
R-squared	0.101	0.125	0.126	0.144	0.122	0.116	0.134

Table A7: This table replicates Table 3 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log \hat{s}_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log \hat{s}_{i,t} + \gamma \log \sigma^e_{i,t} \times \log \hat{s}_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.108***		-0.006	0.056***	0.020	-0.055***	0.209***
	(-7.91)		(-0.54)	(4.31)	(1.34)	(-2.92)	(7.01)
$\log \hat{s}_{i,t}$		-0.282***	-0.275***	-0.135***	-0.222***	-0.283***	-0.255***
		(-13.96)	(-13.88)	(-4.33)	(-8.37)	(-9.15)	(-12.94)
$\log \sigma^e_{i,t} \times \log \hat{s}_{i,t}$							-0.048***
							(-7.19)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	39925	40331	39925	13272	13296	13027	39925
R-squared	0.108	0.140	0.139	0.169	0.145	0.126	0.144

Table A8: This table replicates Table 4 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log s_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \sigma_{i,t}$	0.058***	0.073***	0.039***	0.043***	0.444***
	(6.34)	(6.60)	(3.11)	(3.00)	(7.45)
$\log s_{i,t}$	-0.299***	-0.108***	-0.295***	-0.519***	-0.299***
	(-12.60)	(-2.75)	(-5.98)	(-11.87)	(-12.73)
$\log \sigma_{i,t} \times \log s_{i,t}$					-0.070***
					(-6.54)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	47384	16237	15947	15200	47384
R-squared	0.125	0.149	0.122	0.114	0.128

Table A9: This table replicates Table 7 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B.

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	(1) $\log \sigma^e_{i,t}$	(2) $\log s_{i,t}$	Panel B: Changes	(3) $\Delta \log \sigma^e_{i,t}$	(4) $\Delta \log s_{i,t}$
$\log \sigma_{i,t}$	0.724***	0.109***	$\log \sigma_{i,t}$	0.721***	0.009***
	(90.77)	(15.91)		(83.77)	(5.00)
$\log[MA/ME]_{i,t}$	0.452***	0.407***	$\log[MA/ME]_{i,t}$	0.247***	0.164***
	(56.20)	(32.17)		(25.66)	(24.37)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	47327	47250	Observations	44706	44640
R-squared	0.865	0.571	R-squared	0.794	0.345

Table A10: This table replicates regressions in Table 2 with the same sample used for generating Table 3. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

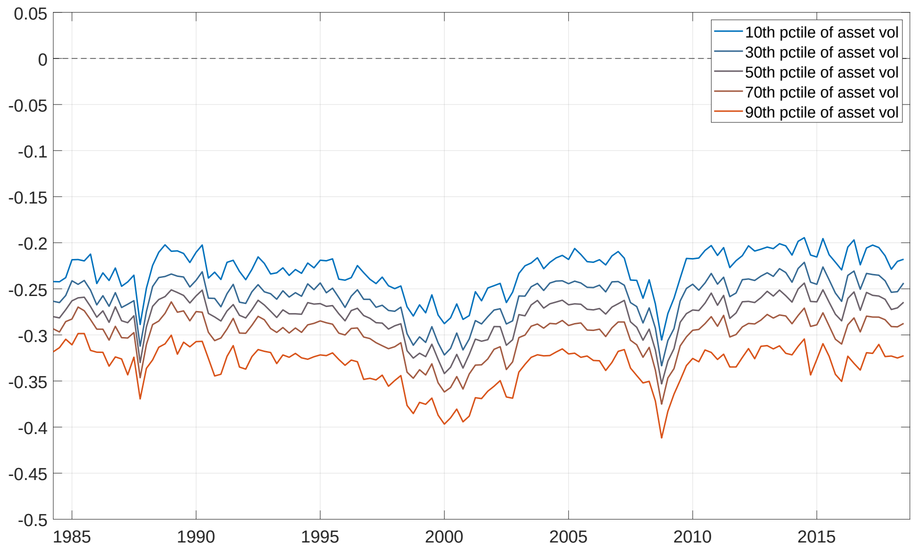
$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.154***		-0.048***	0.059***	-0.014	-0.118***	0.796***
	(-7.91)		(-3.00)	(3.03)	(-0.64)	(-4.35)	(8.98)
$\log s_{i,t}$		-0.278***	-0.262***	-0.111***	-0.290***	-0.392***	-0.465***
		(-11.30)	(-10.81)	(-2.92)	(-5.63)	(-8.68)	(-14.57)
$\log \sigma^e_{i,t} \times \log s_{i,t}$							-0.151***
							(-9.35)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	39925	39970	39595	13272	13296	13027	39595
R-squared	0.108	0.132	0.132	0.165	0.136	0.123	0.140

Figure A1: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log asset volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 4 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.



Notes: This figure is generated using the estimates on $\log s_{i,t}$ and $\log \sigma_{i,t} \times \log s_{i,t}$ in Column 5 of Table 4.

D Proofs

Shareholders maximize their expected cash flow and device when to default. Thus, the value of equity is given by:

$$E = \max_{\iota, \underline{z}} \left\{ \mathbb{E}\Big[(Y(\iota, z) - b)\mathbb{1}\{z \geq \underline{z}\}\Big] - \iota \right\}.$$

We can write the first-order conditions for investment ι and the default boundary $\underline{z}$ as:

$$\begin{aligned} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) dF(z;\sigma) - 1 = 0, \\ -f(\underline{z};\sigma)(Y(\iota, \underline{z}) - b) = 0, \end{aligned}$$

and the second-order conditions for investment ι and the default boundary $\underline{z}$ as:

$$\begin{aligned} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) < 0, \\ -f(\underline{z};\sigma)k(\iota) < 0, \\ -\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) f(\underline{z};\sigma)k(\iota) - f(\underline{z};\sigma)^2 k_\iota(\iota)^2 \underline{z}^2 > 0. \end{aligned} \tag{8}$$

Thus, $Y_\iota(\iota, \underline{z})^2 f(\underline{z};\sigma) + k(\iota)k_{\iota\iota}(\iota)\mu(\underline{z}, \sigma) < 0$.

In the following sections, we derive the partial derivatives of equity with respect to (i) credit spreads and asset volatility, (ii) leverage and asset volatility, and (iii) credit spreads and equity volatility to rationalize our empirical results.

Investment ι, Credit Spreads cs, and Asset Volatility σ Assume we observe $\boldsymbol{\theta}$ and we want to derive the partial derivatives of $\mathbf{x}$ with respect to $\boldsymbol{\theta}$. Since $\mathbf{x}$ is the solution to a system of nonlinear equations $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$, we need to use the multivariate implicit function theorem:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i} \right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) dF(z; \sigma) - 1 \\ F(\underline{z}; \sigma) - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) & -Y_\iota(\iota, \underline{z}) f(\underline{z}; \sigma) \\ 0 & f(\underline{z}; \sigma) \end{bmatrix}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial cs} \right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma} \right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz \\ F_\sigma(\underline{z}) \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}$. Thus, we get:

$$\frac{\partial \iota}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}_{12} = \frac{Y_\iota(\iota, \underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma)} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} < 0,$$

and

$$\begin{aligned} \frac{\partial \iota}{\partial \sigma} &= - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}_{11} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}_{12} F_\sigma(\underline{z}) \\ &= - \frac{\int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz + Y_\iota(\iota, \underline{z}) F_\sigma(\underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma)} \\ &= - \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)} > 0. \end{aligned}$$

The sign of both these partial derivatives comes directly from Assumptions 5.4 and 5.5.

Investment ι, Leverage b, and Asset Volatility σ Instead of observing credit spreads cs and asset volatility, we observe leverage b and asset volatility σ. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) dF(z; \sigma) - 1 \\ Y(\iota, \underline{z}) - b \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} b & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) & -Y_\iota(\iota, \underline{z}) f(\underline{z}; \sigma) \\ Y_\iota(\iota, \underline{z}) & k(\iota) \end{bmatrix}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial b}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz \\ 0 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive:

$$\begin{aligned} \frac{\partial \iota}{\partial b} &= \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \\ &= \frac{Y_\iota(\iota, \underline{z}) f(\underline{z}; \sigma)}{Y_\iota(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)} < 0, \end{aligned}$$

$$\frac{\partial \iota}{\partial \sigma} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz$$
$$= -\frac{k(\iota) \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz}{Y_\iota(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)},$$

$$\frac{\partial \underline{z}}{\partial b} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{22}$$
$$= -\frac{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma)}{Y_\iota(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)} > 0,$$

$$\frac{\partial \underline{z}}{\partial \sigma} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{21} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz$$
$$= \frac{Y_\iota(\iota, z) \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz}{Y_\iota(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)}.$$

The sign of these partial derivatives comes directly from Assumptions 5.4 and 5.5 and the second-order condition in equation (8). Furthermore, we can derive:

$$\frac{\partial cs}{\partial \sigma} = f(\underline{z}; \sigma) \frac{\partial \underline{z}}{\partial \sigma} + F_\sigma(\underline{z}; \sigma).$$

Investment ι, Credit Spreads cs, and Equity Volatility σ^e Instead of observing credit spreads cs and asset volatility, we observe credit spreads cs and equity volatility σ^e. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)dF(z;\sigma) - 1 \\ F(\underline{z};\sigma) - cs \\ \frac{\sigma}{\mu(\underline{z},\sigma)} - \sigma^e \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \\ \sigma \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma^e \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma) & -Y_\iota(\iota,\underline{z})f(\underline{z};\sigma) & \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz \\ 0 & f(\underline{z};\sigma) & F_\sigma(\underline{z}) \\ 0 & \sigma^e_{\underline{z}} & \sigma^e_\sigma \end{bmatrix},$$

where

$$\begin{aligned} \sigma^e_{\underline{z}} &= -\frac{\sigma\mu_{\underline{z}}(\underline{z},\sigma)}{\mu(\underline{z},\sigma)^2} = \frac{\sigma\left(1 - F(\underline{z};\sigma)\right)}{\mu(\underline{z},\sigma)^2}, \\ \sigma^e_\sigma &= \frac{\mu(\underline{z},\sigma) - \sigma\nu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)^2}, \end{aligned}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma^e}\right] = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of

$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive:

$$\begin{aligned}
\frac{\partial \iota}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} &= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)\mu(\underline{z},\sigma)} \frac{\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} z f_\sigma(z;\sigma)dz + \sigma^e_\sigma(\underline{z},\sigma)\underline{z}f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z},\sigma)} \frac{\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} \frac{z}{\underline{z}}\frac{f_\sigma(z;\sigma)}{f(\underline{z};\sigma)}dz + \sigma^e_\sigma(\underline{z},\sigma)}{\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z})}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z},\sigma)} \left(\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} \frac{z}{\underline{z}}\frac{f_\sigma(z;\sigma)}{f(\underline{z};\sigma)}dz + \sigma^e_\sigma(\underline{z},\sigma)\right) \\
&\times \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \iota}{\partial \sigma^e} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{13} &= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)\mu(\underline{z},\sigma)} \frac{f(\underline{z};\sigma)\underline{z}F_\sigma(\underline{z};\sigma) + f(\underline{z};\sigma)\int_{\underline{z}}^{\infty} z f_\sigma(z;\sigma)dz}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)} \\
&= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z})}{\mu(\underline{z},\sigma)} \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.
\end{aligned}$$

We define the the credit spread wedge as:

$$\begin{aligned}
\xi_{cs|\sigma^e}(\underline{z},\sigma) &= \left(\sigma^e_{\underline{z}}(\underline{z},\sigma)\left(\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z};\sigma)} - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\right) + \sigma^e_\sigma(\underline{z},\sigma)\right)\zeta^e(\underline{z},\sigma) \\
&= 1 + \sigma^e_{\underline{z}}(\underline{z},\sigma)\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z};\sigma)}\zeta^e(\underline{z},\sigma),
\end{aligned}$$

and the equity volatility wedge as:

$$\xi_{\sigma^e|cs}(\underline{z},\sigma) = \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.$$

Investment ι, Credit Spreads with Positive Liquidation Value $\widetilde{cs}$, and Asset Volatility σ Instead of observing credit spreads cs and asset volatility σ, we observe credit spreads cs with positive liquidation value $\widetilde{cs}$ and asset volatility σ.

We define the credit spreads with positive liquidation value as:

$$\widetilde{cs} = F(\underline{z};\sigma) - \frac{\alpha}{b}\mathbb{E}\Big[Y(\iota,z)\mathbb{1}\{z \le \underline{z}\}\Big].$$

where $1-\alpha$ represents bankruptcy costs. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)dF(z;\sigma) - 1 \\ F(\underline{z};\sigma) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)dF(z;\sigma) - \widetilde{cs} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} \widetilde{cs} & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma) & -Y_\iota(\iota,\underline{z})f(\underline{z};\sigma) \\ -\frac{\alpha}{b}\int_0^{\underline{z}} Y_\iota(\iota,z)dF(z;\sigma) & f(\underline{z};\sigma)(1-\alpha) \end{bmatrix}$$

since $Y(\iota,\underline{z}) = b$, and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \widetilde{cs}}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz \\ F_\sigma(\underline{z}) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)f_\sigma(z)dz \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we get:

$$\begin{aligned}
\frac{\partial \iota}{\partial cs} &= \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = \frac{Y_\iota(\iota,\underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)\,(1-\alpha) - \frac{\alpha}{b}Y_\iota(\iota,\underline{z})\int_0^{\underline{z}} Y_\iota(\iota,z)dF(z;\sigma)} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\underline{z}}{\mu(\underline{z},\sigma)\,(1-\alpha) - \alpha\frac{k_\iota(\iota)}{k(\iota)}\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}(1-\mu(\underline{z},\sigma))} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\underline{z}}{\mu(\underline{z},\sigma)}\frac{1}{(1-\alpha) - \alpha\frac{k_\iota(\iota)}{k(\iota)}\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{1-\mu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)}} < 0.
\end{aligned}$$

Indeed, since $\alpha \leq 0$, we get that $1 - \alpha \geq 0$. Furthermore,

$$\begin{aligned}
\frac{\partial \iota}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \left(F_\sigma(\underline{z}) - \frac{\alpha}{b} \int_0^{\underline{z}} Y(\iota, z) f_\sigma(z) dz\right) \\
&= -\frac{\int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz \, (1-\alpha) + Y_\iota(\iota, \underline{z}) \left(F_\sigma(\underline{z}) - \frac{\alpha}{b} \int_0^{\underline{z}} Y(\iota, z) f_\sigma(z) dz\right)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) \, (1-\alpha) - \frac{\alpha}{b} Y_\iota(\iota, \underline{z}) \int_0^{\underline{z}} Y_\iota(\iota, z) dF(z;\sigma)} \\
&= -\frac{k_\iota(\iota) \int_{\underline{z}}^{\infty} z f_\sigma(z) dz \, (1-\alpha) + k_\iota(\iota) \left(\underline{z} F_\sigma(\underline{z}) - \alpha \int_0^{\underline{z}} z f_\sigma(z) dz\right)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) \, (1-\alpha) - \alpha \frac{k_\iota(\iota)^2}{k(\iota)} (1 - \mu(\underline{z}, \sigma))} \\
&= -\frac{k_\iota(\iota)(\nu(\underline{z}, \sigma) - \alpha \int_0^{\infty} z f_\sigma(z) dz)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) \, (1-\alpha) - \alpha \frac{k_\iota(\iota)^2}{k(\iota)} (1 - \mu(\underline{z}, \sigma))} \\
&= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)} \frac{1}{(1-\alpha) - \alpha \frac{k_\iota(\iota)}{k(\iota)} \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{1 - \mu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)}} > 0.
\end{aligned}$$

Indeed, $1 - \mu(\underline{z}, \sigma) \geq 0$.

E Endogenous Leverage Dynamics

In this appendix, we extend the framework of DeMarzo and He (2020) to include an investment function. We solve numerically the Markov perfect equilibrium and confirm that our results hold in Figure 2. We refer to DeMarzo and He (2020) for the proofs of the existence and uniqueness of the Markov perfect equilibrium.

We assume that agents are risk neutral with an exogenous discount rate of $r > 0$. The firm's assets-in-place generate operating cash flow at the rate of Y_t, which evolves according to a geometric Brownian motion:

$$dY_t/Y_t = \mu_t dt + \sigma dZ_t$$

where Z_t is a standard Brownian motion. A firm has at its disposal an investment technology with adjustment costs, such that $\iota_t Y_t$ spent allows the firm to grow its capital stock by $\mu(\iota_t)Y_t dt$, where $\mu(\cdot)$ is increasing and concave. Denote by B the aggregate face value of outstanding debt that pays a constant coupon rate of $c > 0$. The firm pays corporate taxes equal to $\pi(Y_t - cF_t)$. We assume that debt takes the form of exponentially maturing coupon bonds with a constant amortization rate ξ. Equity holders control the outstanding debt B_t through an endogenous issuance/repurchase policy $d\Gamma_t$ but cannot commit on a policy. Thus, the evolution of the outstanding face value of debt follows

$$dB_t = d\Gamma_t - \xi B_t dt.$$

In the unique Markov equilibrium, given the debt price $p(Y, B)$, the firm's issuance policy $d\Gamma_t = G_t dt$ and default time τ maximize the market value of equity:

$$E(Y, B) = \max_{\tau, \iota_t, G_t} \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} \big[(1 - \iota_s)Y_s - \pi(Y_s - cB_s) - (c + \xi)B_s + G_s p_s \big] ds \,\Big|\, Y_t = Y, B_t = B \right].$$

Similarly, the equilibrium market price of debt must satisfy

$$p(Y,B) = \mathbb{E}_t\left[\int_t^\tau e^{-(r+\xi)}(c+\xi)ds \middle| Y_t = Y, B_t = B\right].$$

The Hamilton-Jacobi-Bellman (HJB) equation for equity holders is

$$rE(Y,B) = \max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s \\ + Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B) + \frac{1}{2}\sigma^2Y^2E_{YY}(Y,B)\Big]. \tag{9}$$

Thus, in equilibrium it must be that

$$p(Y,B) = -E_B(Y,B).$$

The first-order condition for the investment rate is given by

$$1 = \mu_\iota(\iota)E_Y(Y,B).$$

In the following, we define $\{\iota(Y,B), G(Y,B)\}$ as

$$\{\iota(Y,B), G(Y,B)\} = \arg\max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s \\ + Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B) \\ + \frac{1}{2}\sigma^2Y^2E_{YY}(Y,B)\Big].$$

In this setting with scale-invariance, the relevant measure of leverage is given by

$$y_t \equiv Y_t/B_t,$$

and the equity value function $E(Y,B)$ and debt price $p(Y,B)$ satisfy

$$E(Y,B) = E\left(Y/B,1\right) \equiv e(y)B \quad \text{and} \quad p(Y,B) = p(Y/B,1) \equiv p(y).$$

We also define the following:

$$\iota(Y,B) \equiv \iota(y) \quad \text{and} \quad G(Y,B) \equiv g(y)B.$$

Thus, we can rewrite (9) as follows

$$(r+\xi)e(y) = \max_{\iota}\left[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)ye'(y) + \frac{1}{2}\sigma^2 y^2 e''(y)\right]. \tag{10}$$

The optimal default boundary is such that

$$e'(y_b) = 0.$$

The higher bound is such that

$$e'(y) = \phi y - \rho,$$

which corresponds to the value of equity without a default option. We can solve for ϕ and ρ with

$$(r+\xi)(\phi y - \rho) = \max_{\iota}\left[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)\phi y\right].$$

Thus,

$$\begin{aligned}
\rho &= \frac{(1-\tau)c+\xi}{r+\xi}, \\
\phi &= \frac{1-\iota^\star-\pi}{r-\mu(\iota^\star)}, \\
1 &= \mu'(\iota^\star)\phi.
\end{aligned}$$

The HJB for $p(Y,B)$ is given by

$$rp(Y,B) = c + \xi(1-p(Y,B)) + (G-\xi B)p_B(Y,B) + \mu(Y,B)Yp_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 p_{YY}(Y,B).$$

where we define $\mu(Y,B) \equiv \mu(\iota(Y,B)) \equiv \mu(y)$.

Thus, we can write the HJB for $p(y)$ as

$$rp(y) = c + \xi(1-p(y)) - (g(y)-\xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y). \tag{11}$$

where $g(y) = G(Y,B)/B$. We need $g(y)$ to be such that $p(y) = e'(y)y - e(y)$. From (10), we get

$$\begin{aligned}(r+\xi)e'(y)y = (1-\iota(y))y - \pi y - \iota'(y)y^2 + (\mu(y)+\xi)y^2e''(y) + (\mu(y)+\xi)ye'(y) + \mu'(y)y^2e'(y) \\ + \frac{1}{2}\sigma^2 y^3 e'''(y)) + \sigma^2 y^2 e''(y).\end{aligned}$$

Thus,

$$\begin{aligned}(r+\xi)(e'(y)y - e(y)) = (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)ye''(y) + \mu'(y)y^2e'(y) \\ + \frac{1}{2}\sigma^2 y^2 e'''(y) + \frac{1}{2}\sigma^2 y^2 e''(y).\end{aligned}$$

Thus, $g(y)$ is such that

$$\begin{aligned}&c + \xi - (g(y)-\xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \\ &= (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)y^2e''(y) + \mu'(y)y^2e'(y) \\ &+ \frac{1}{2}\sigma^2 y^3 e'''(y)) + \frac{1}{2}\sigma^2 y^2 e''(y).\end{aligned}$$

With further algebra, we get

$$-gp'(y)y = -\pi c - \iota'(y)y^2 + \mu'(y)y^2e'(y).$$

Since $\mu'(\iota)e'(y) = 1$ and $\mu'(y) = \mu'(\iota)\iota'(y)$, we get

$$g(y) = \frac{\pi c}{p'(y)y}.$$

Plugging the solution for $g(y)$ in (11) yields

$$(r+\xi)p(y) = (1-\pi)c + \xi + (\mu(y)+\xi)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y).$$

We solve numerically for the solution using ODE45 in Matlab. We use the following pseudo-algorithm.

1. Start with $y_L = 0$ and $y_H = H$, where H is a sufficiently large number.

2. Given $y_b = 1/2(y_L + y_H)$, $e(y_b) = 0$, and $e'(y_b) = 0$, we solve for $e(y)$ on $[y_b, y_B]$ where Y_B is a large number.

3. Check if $|e(Y_B) - (\phi y_B - \rho)| \leq \varepsilon$, where $\varepsilon > 0$ is a small number. If $e(Y_B) - (\phi y_B - \rho) > \varepsilon$, set $y_L = y_b$ and repeat 2-3. If $e(y_B) - (\phi y_B - \rho) < -\varepsilon$, set $y_H = y_b$ and repeat 2-3. Otherwise move to 4.

4. Start with $pp_L = 0$ and $pp_H = H$, where H is a sufficiently large number.

5. Given $pp_b = 1/2(pp_L + pp_H)$, $p(y_b) = 0$, $p'(y_b) = pp_b$ we solve for $p(y)$ on $[y_b, y_B]$.

6. Check if $|p(y_B) - \rho| \leq \varepsilon$ If $p(Y_B) - \rho > \varepsilon$, set $p_H = p_b$ and repeat 2-3. If $p(y_B) - \rho < -\varepsilon$, set $pp_L = pp_b$ and repeat 4-5. Otherwise move to 7.

7. Check if $|p'(y_b) - e''(y_b)y_b| \leq \varepsilon$. If not, increase the precision of the ODE45 solver and restart from 1.

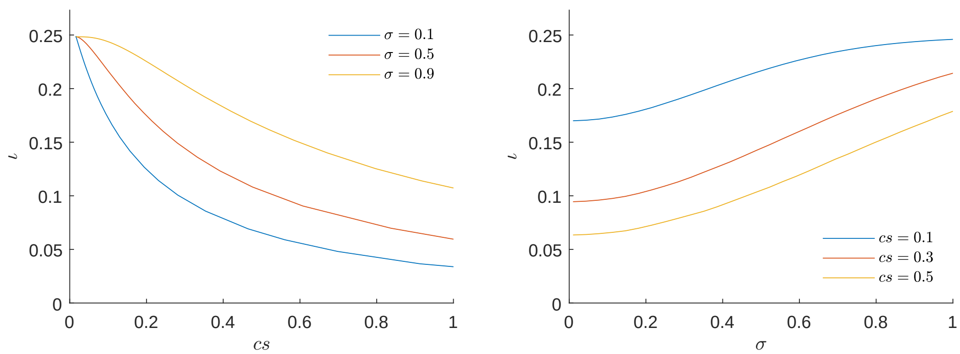


Figure 2: Optimal investment in dynamic setting with $\mu(\iota) = \frac{\log(1+\kappa\iota)}{\kappa}$, $\kappa = 100$, $r = 0.05$, $\xi = 1/8$, $c = 0.05$, $\pi = 0.3$.